



Time-Reversed Waves and Subwavelength Focusing

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Ultrasound, Microwaves and Optics: The Source/Detector Problem

Ultrasound



Microwave



Each antenna (ultrasound or microwave) is able to record the temporal modulation of the wave field and to emit any temporal waveform.

Optics

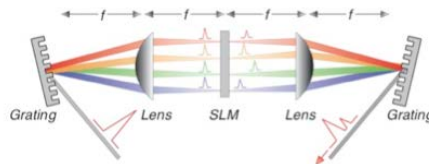
Spatial shaping



Liquid crystals SLM

Optics

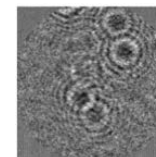
Temporal shaping



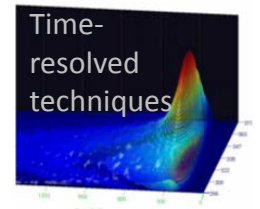
Femtosecond Coherent control

Optics

Interferometry/detection

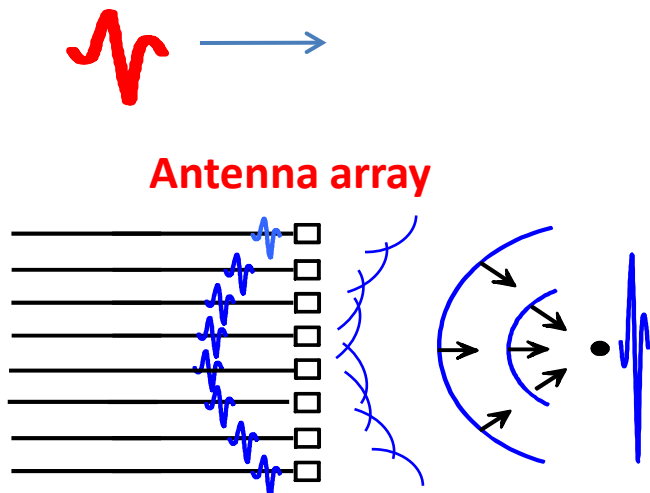
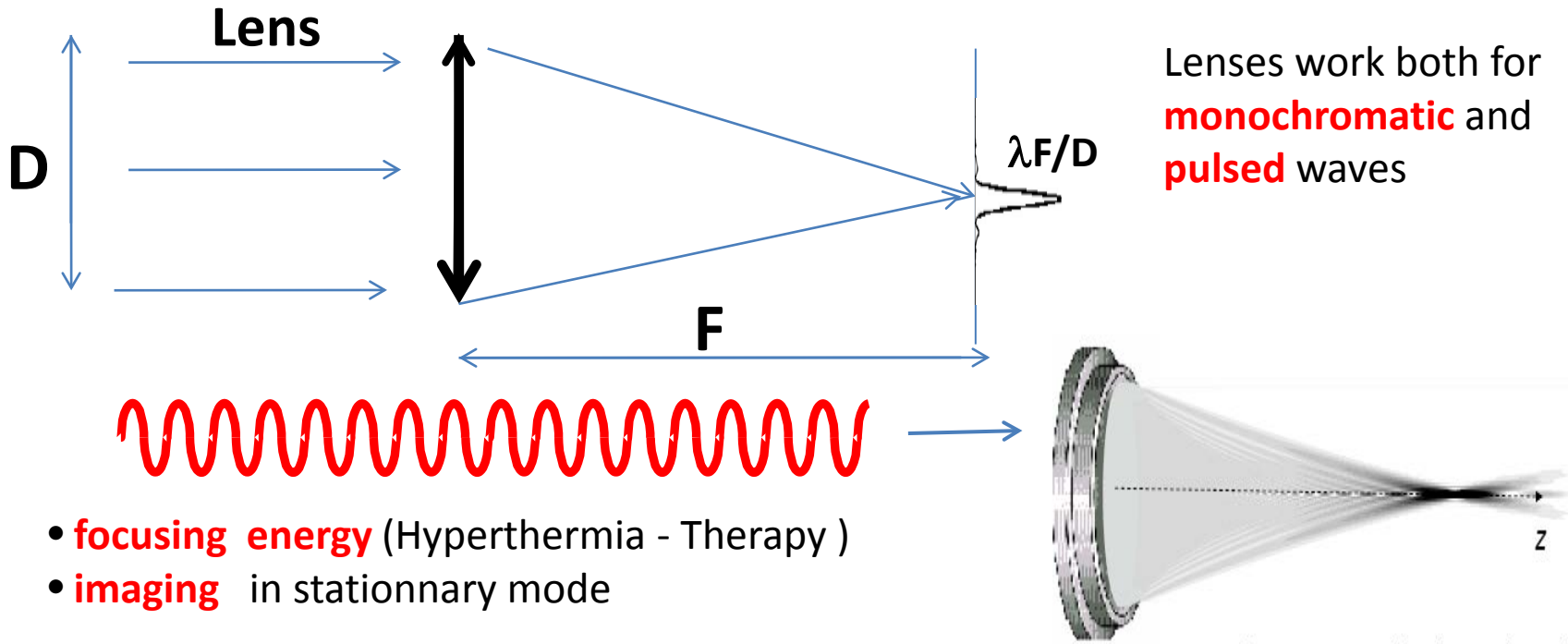


holography



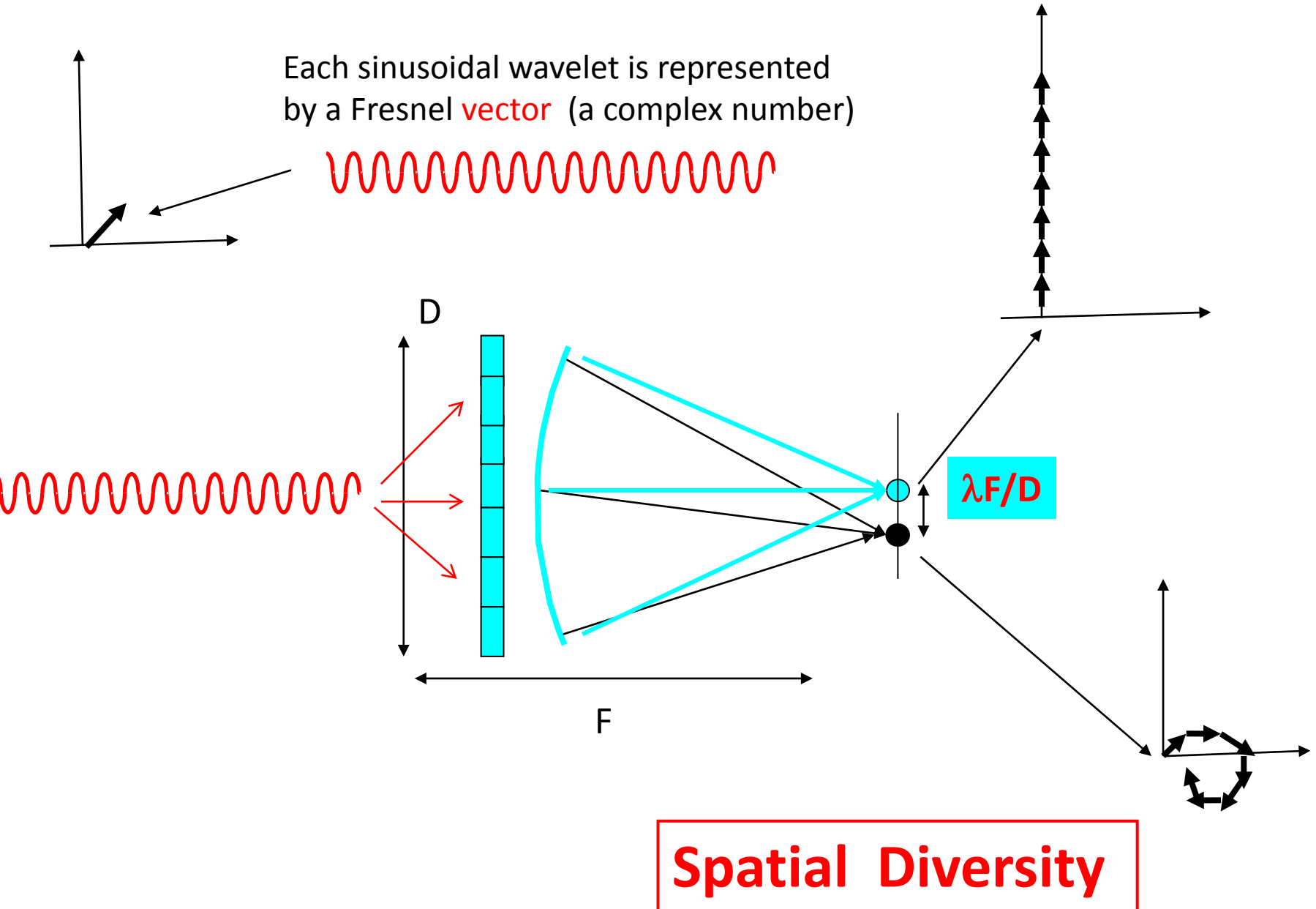
Time-resolved techniques

Wave Focusing in homogeneous medium

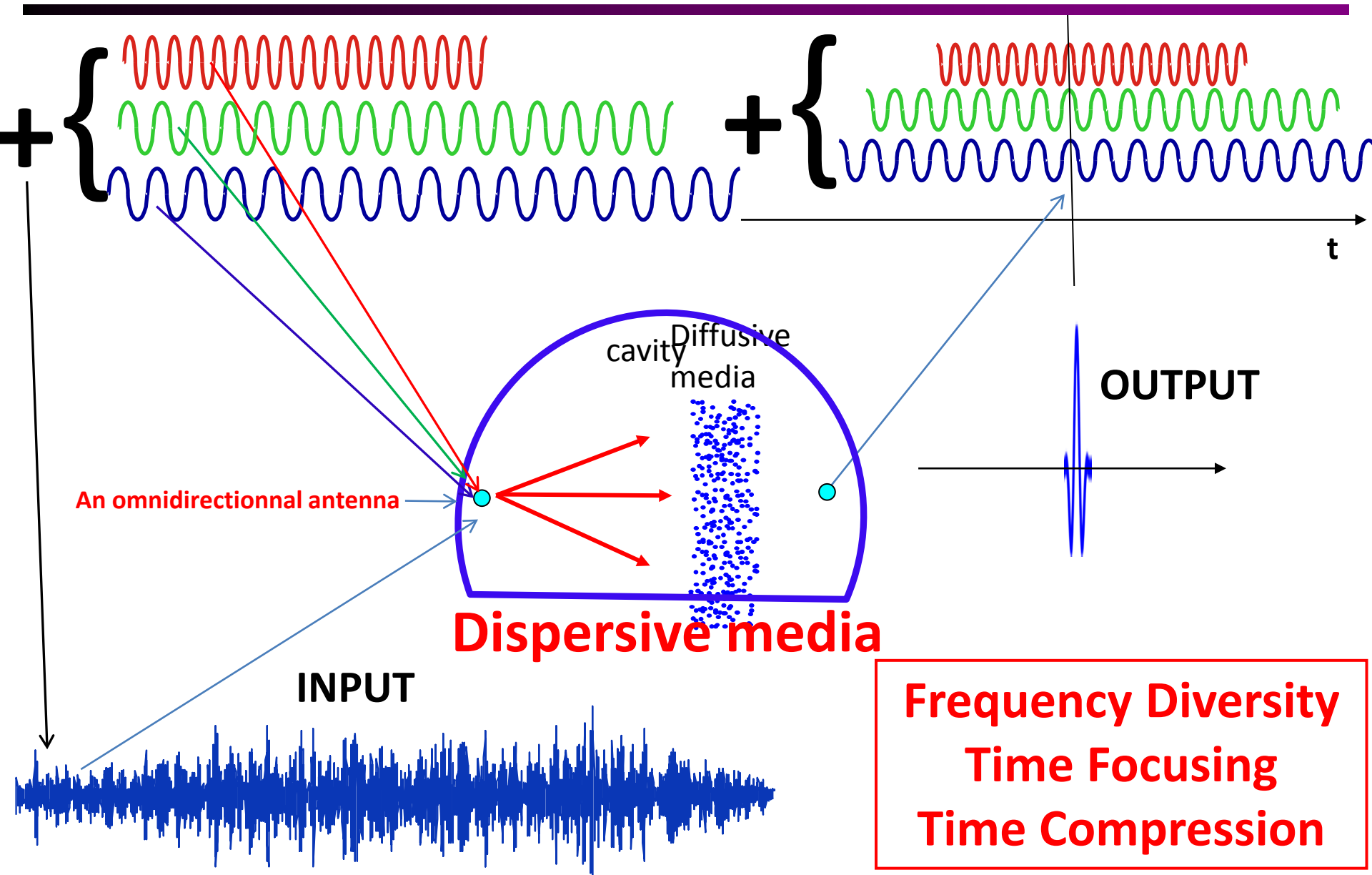


- **Imaging** in pulsed mode (radar, sonar)
- **Focusing shock waves or electromagnetic pulses** (therapy or defense)
- **Telecommunications** discrete broadband telecommunications

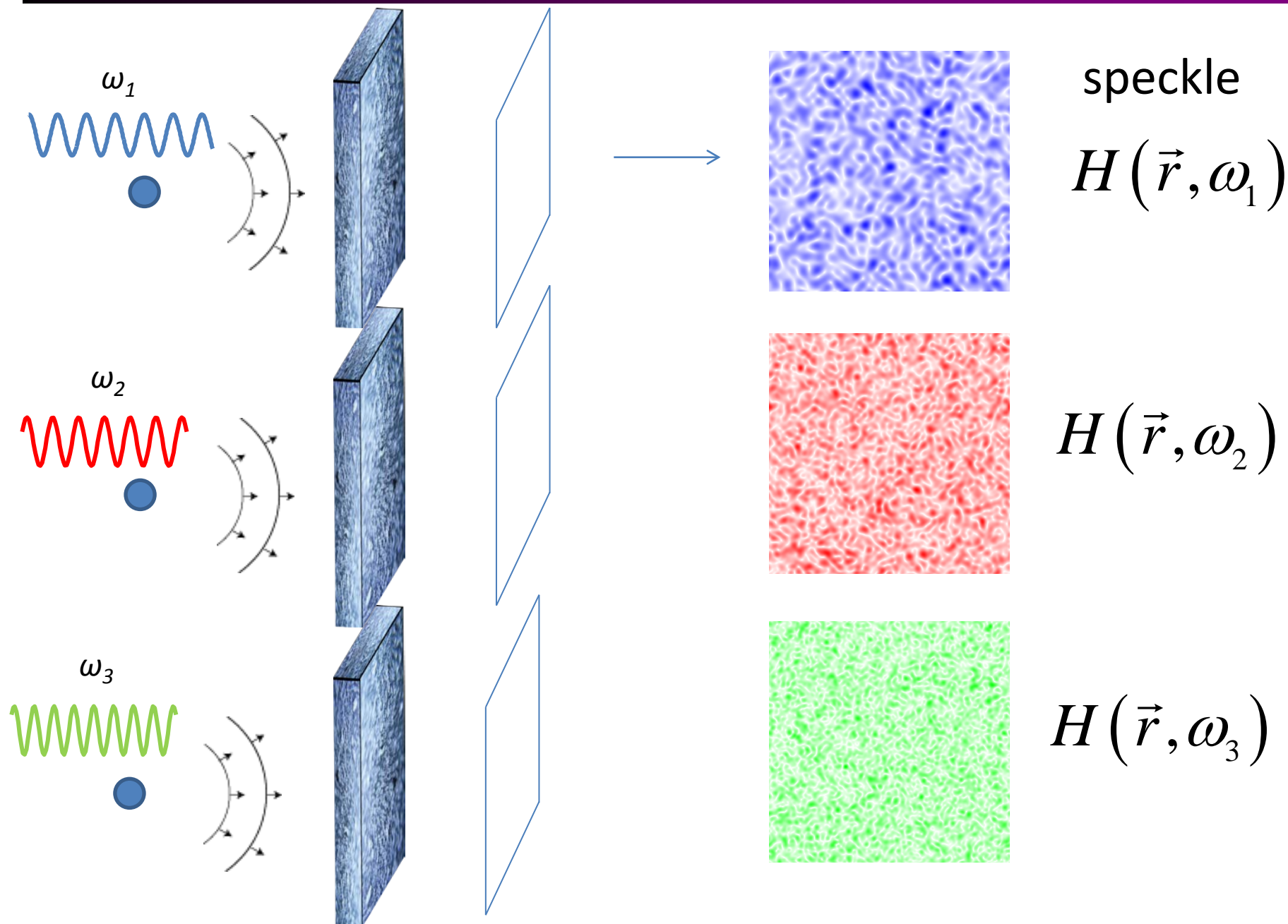
Focusing a monochromatic wave



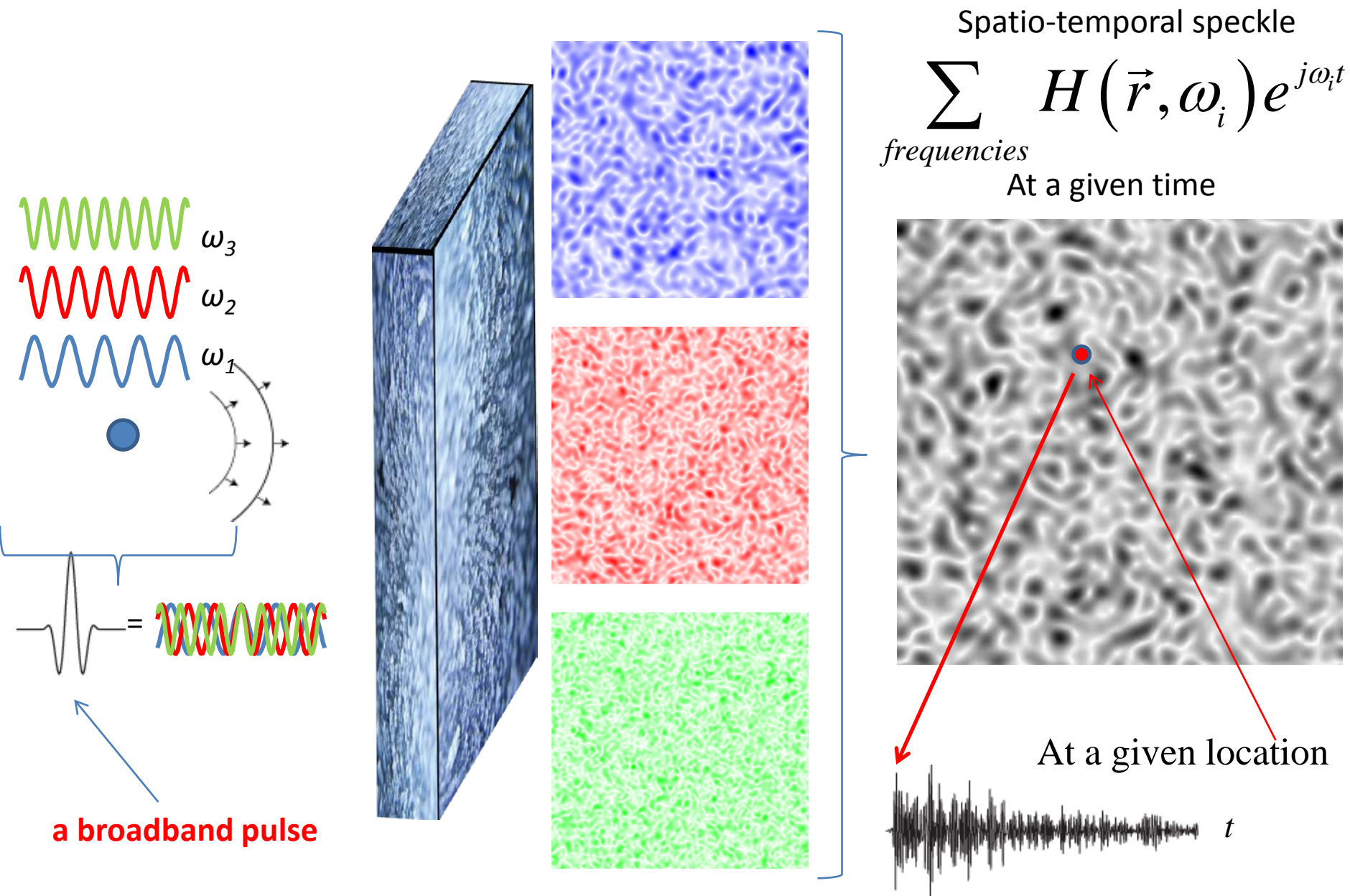
Focusing a Polychromatic Wave through dispersive media



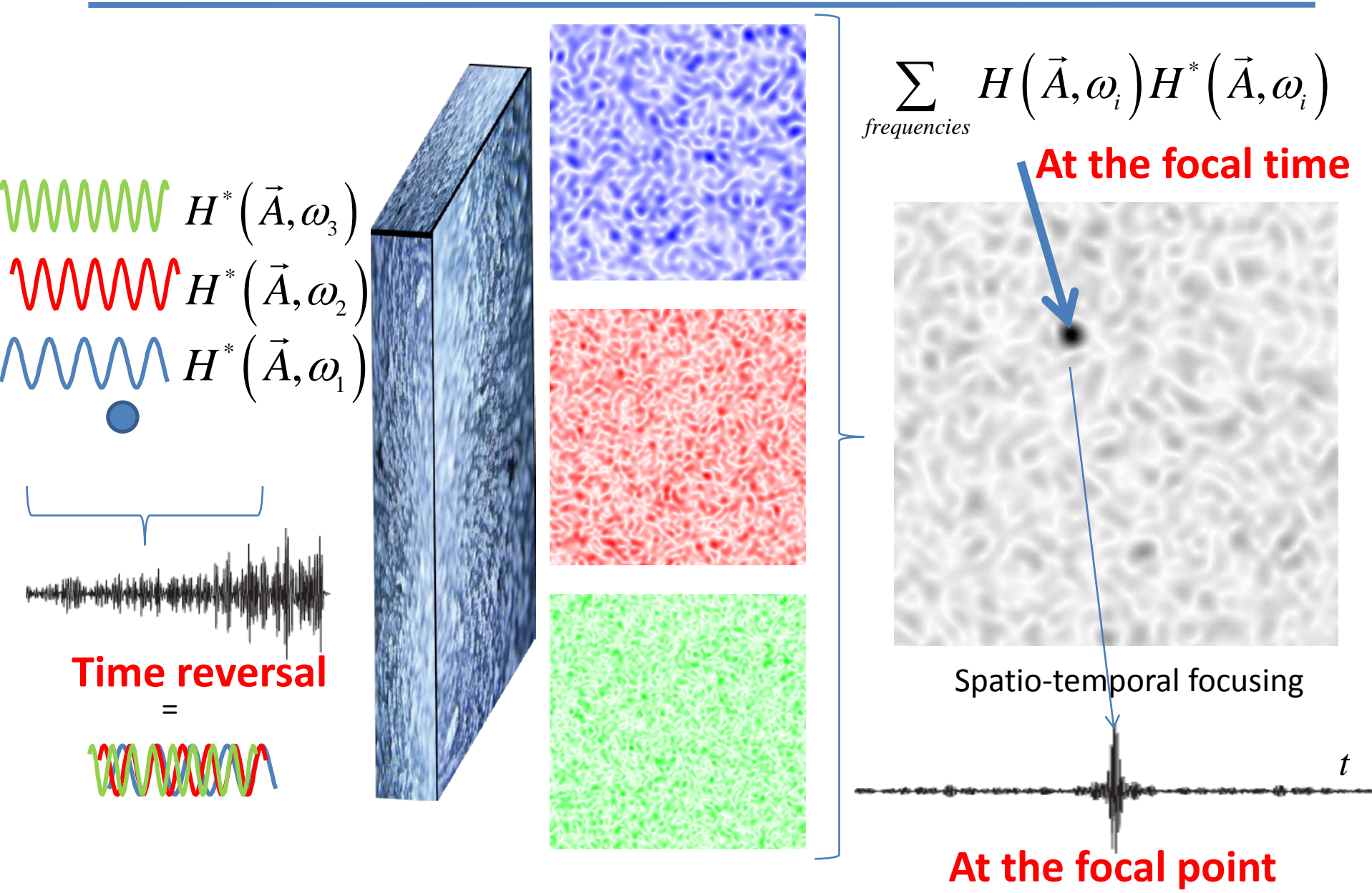
Spatial focusing with temporal degrees of freedom (frequential)



Spatial focusing with temporal degrees of freedom (frequential)



Phase Conjugation and Time Reversal



**Focusing quality and diffraction limits:
the connection with time-reversed
waves**

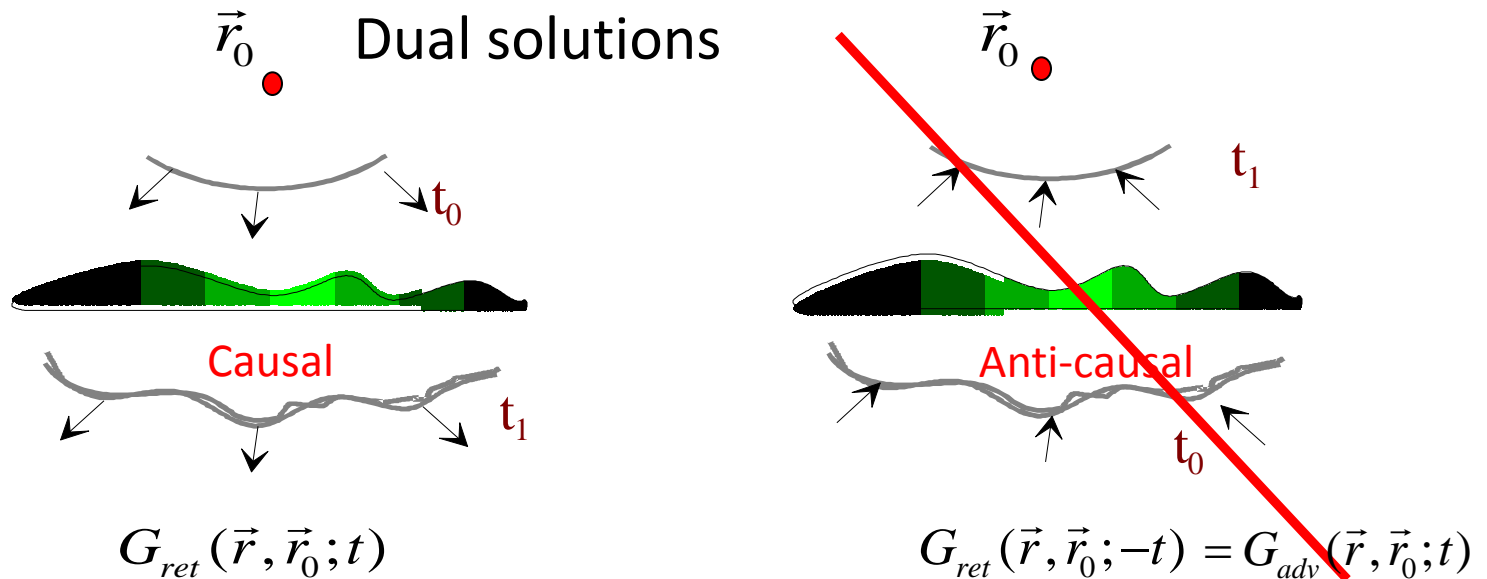
Focusing a broadband wave in an heterogeneous medium

A solution : building a time-reversed wave (causality Pb)

The acoustic case : **non dissipative heterogeneous** medium with a ponctual source

$$\left\{ \Delta - \frac{1}{c^2(\vec{r})} \frac{\partial^2}{\partial t^2} \right\} G(\vec{r}, \vec{r}_0; t) = -\delta(\vec{r} - \vec{r}_0) \delta(t)$$

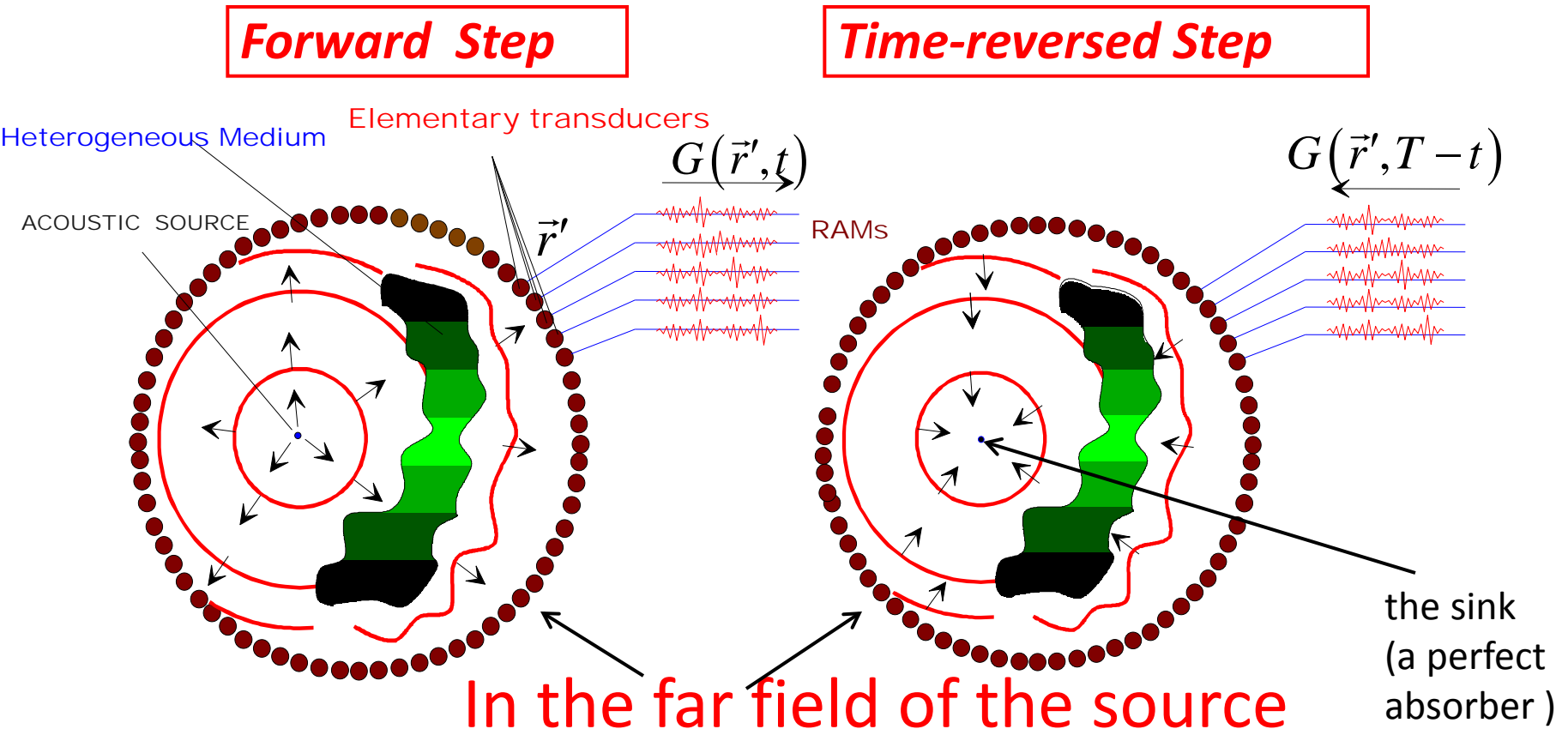
Green's function



To build the TR field : Time reversed the causal field on the boundary : the time reversal cavity

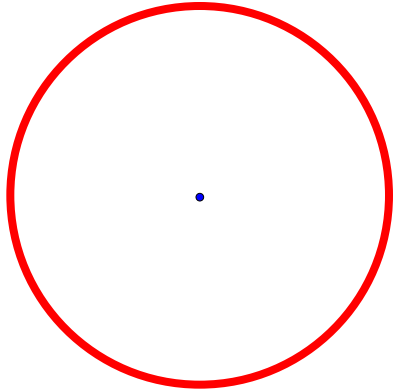
TR on the boundary : the TR Cavity

- record on the boundary $G(\vec{r}', \vec{r}_0; t); \partial_n G(\vec{r}', \vec{r}_0; t)$
- transmit from the boundary $G(\vec{r}', \vec{r}_0; T - t); \partial_n G(\vec{r}', \vec{r}_0; T - t)$

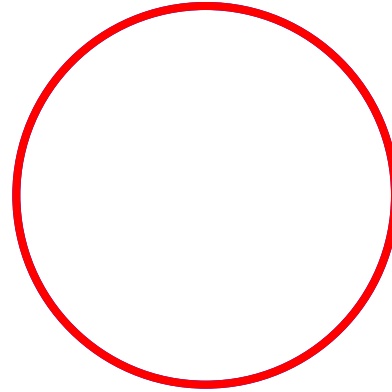


What is the focal spot size ?

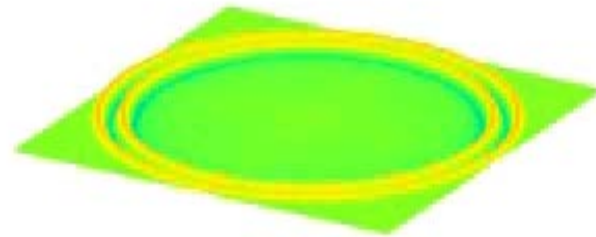
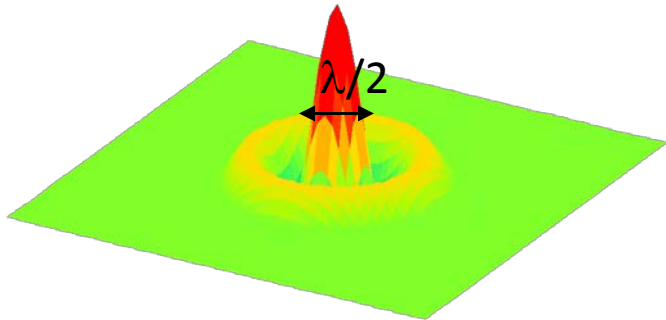
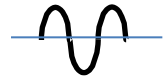
Initial step



Time-reversed step

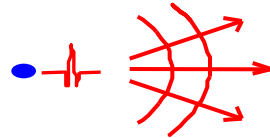


wavelength λ

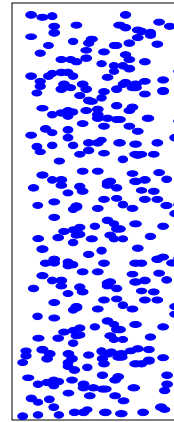


Finite aperture TR through a diffusive medium

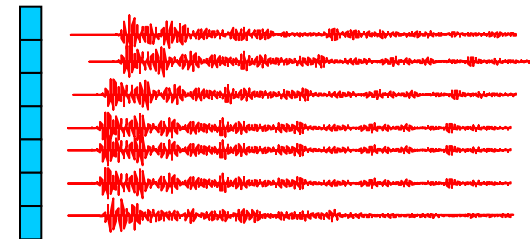
Source



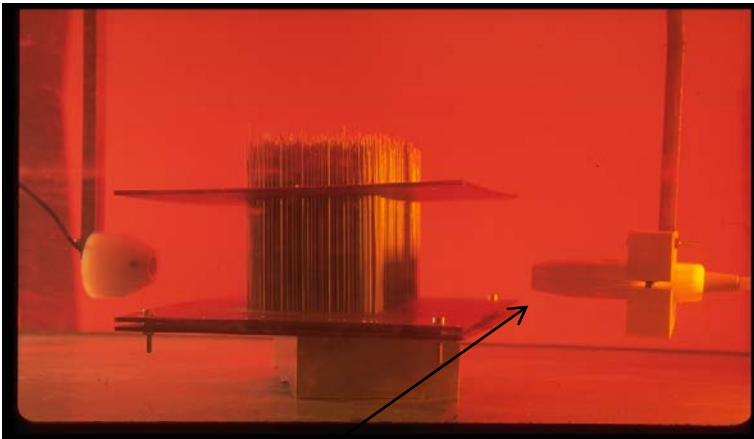
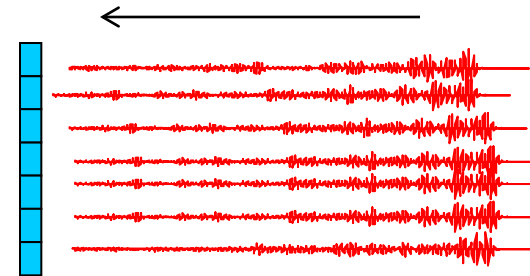
Multiple scattering



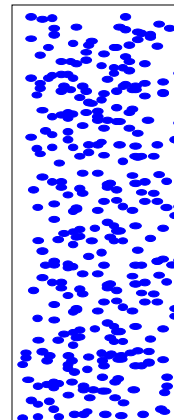
TRM array



Time reversed signals

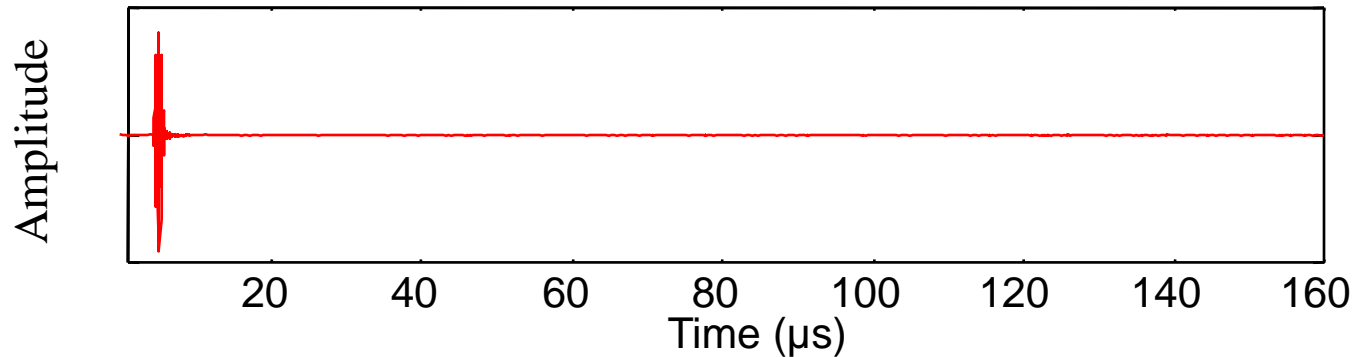


An array of 128 ultrasonic transducers

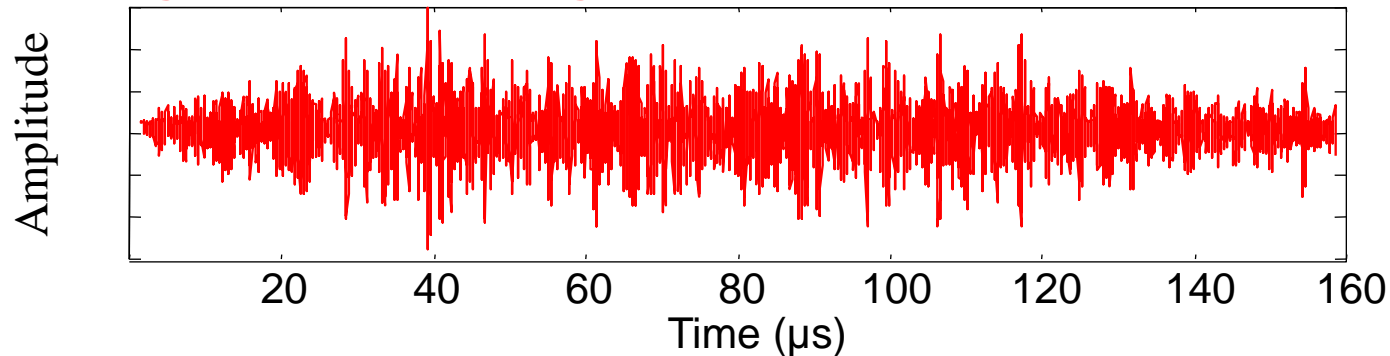


Time Focusing

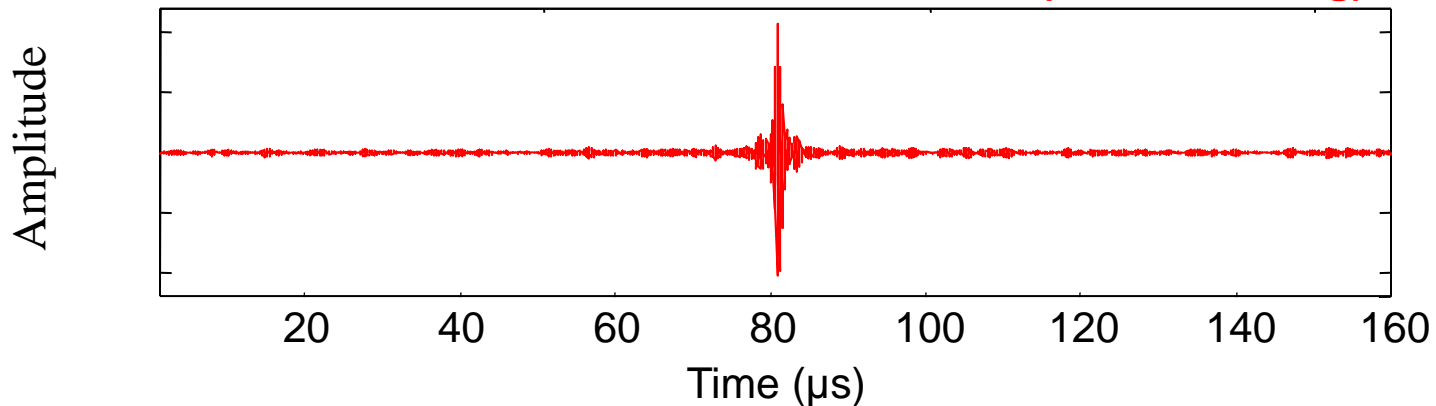
Transmitted signal



Signal recorded through the diffusive medium on transducer 64

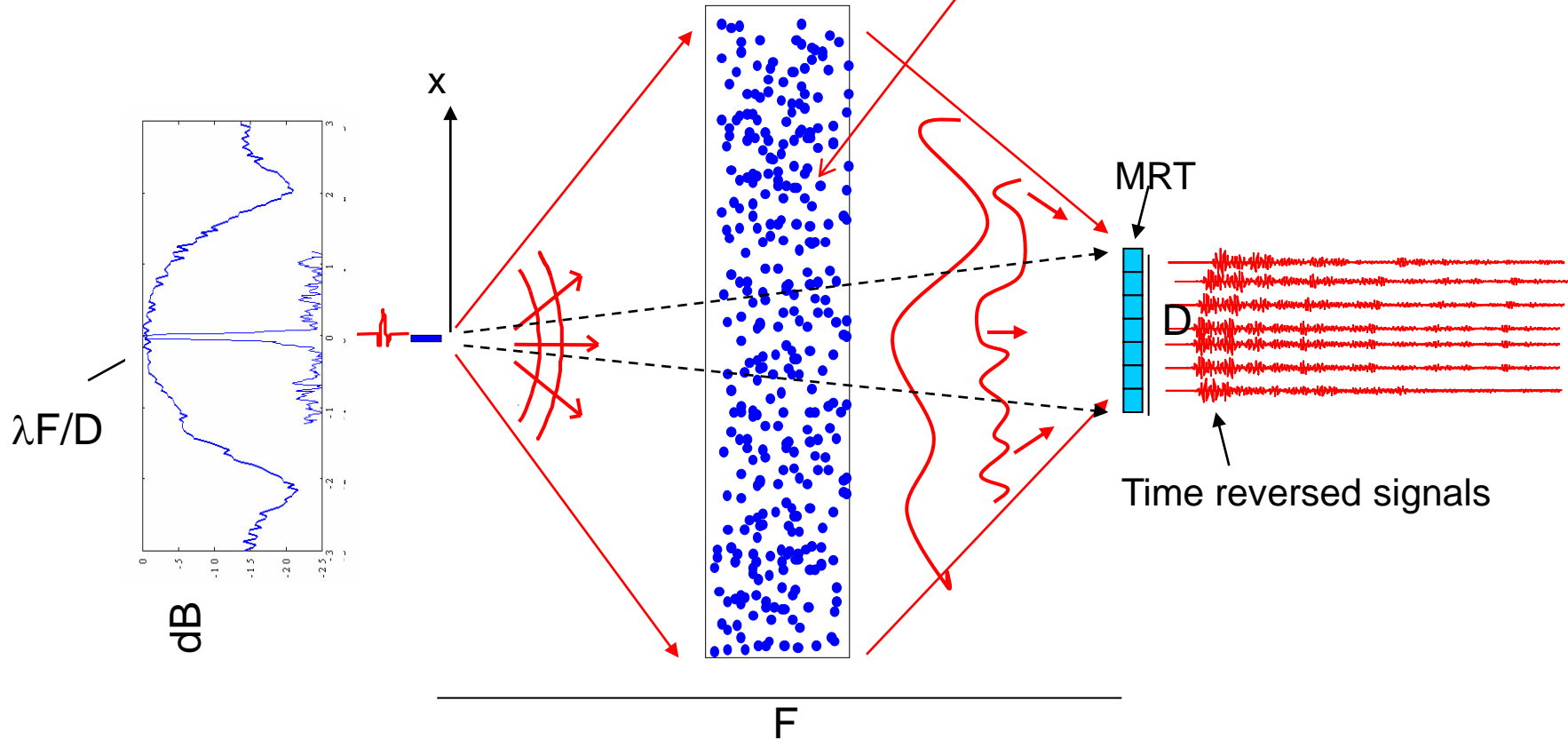


Time reversed wave at the source location (Time Focusing)



Spatial Focusing

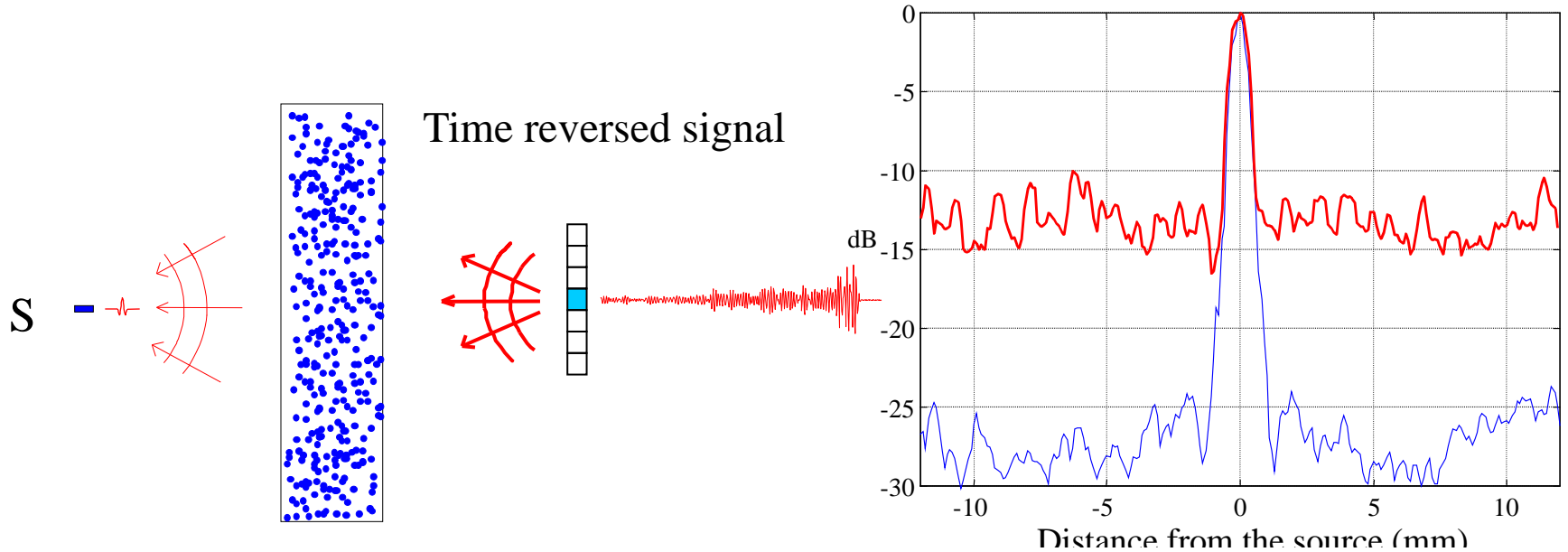
The medium is the Lens



Focal spot : beamwidth at -6 dB : **35 mm / 1 mm**

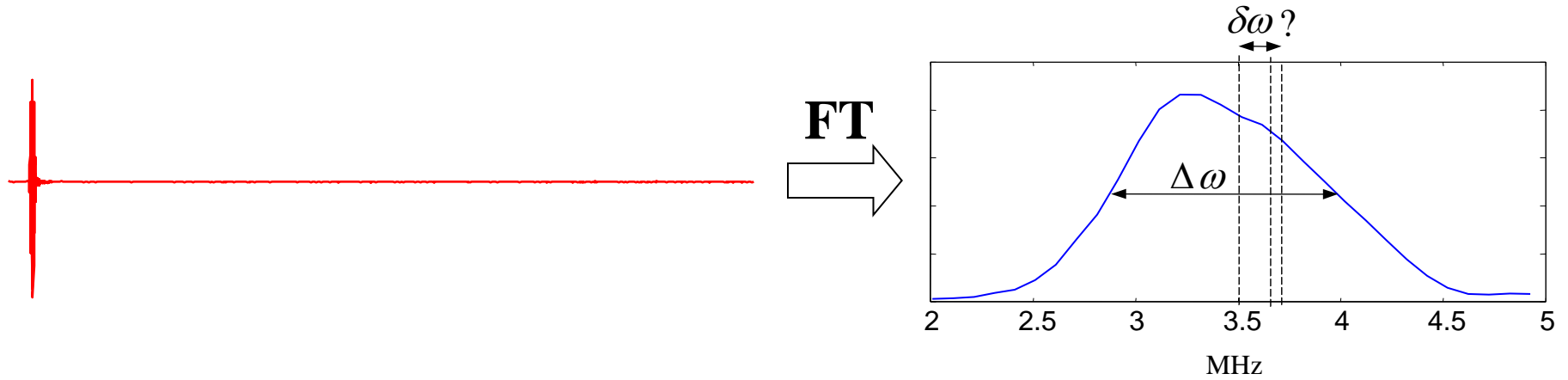
Spatial resolution does not depend on the array aperture !!!

One channel time reversal mirror

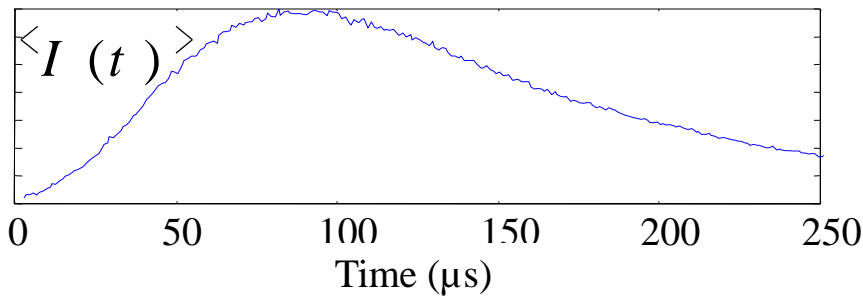
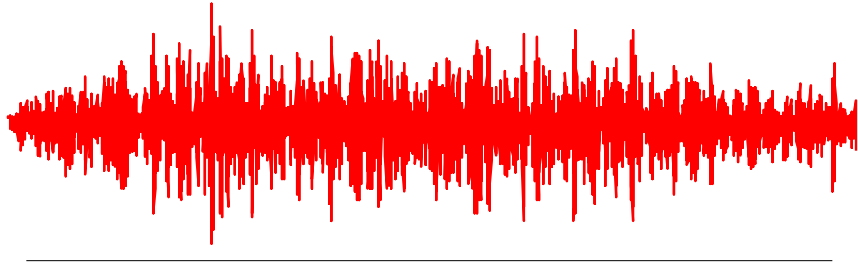


Directivity patterns of the time-reversed waves around the source position with 128 transducers (blue line) and 1 transducer (red line).

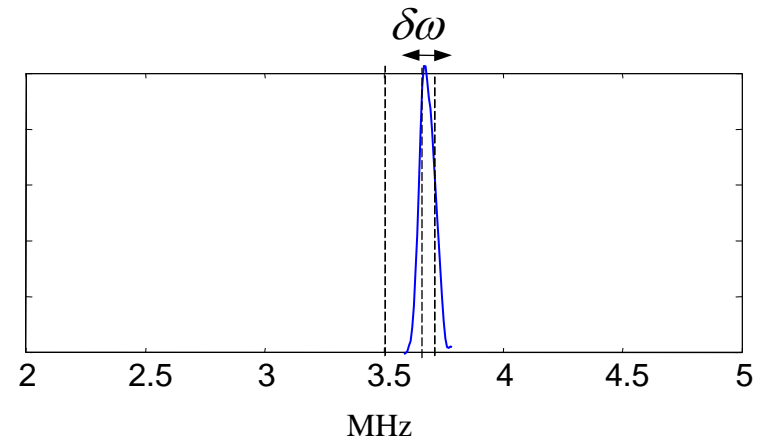
How many uncorrelated speckles ? (temporal degrees of freedom)



Spectral Field-field correlation = fourier transform of the travel time distribution $\langle I(t) \rangle$



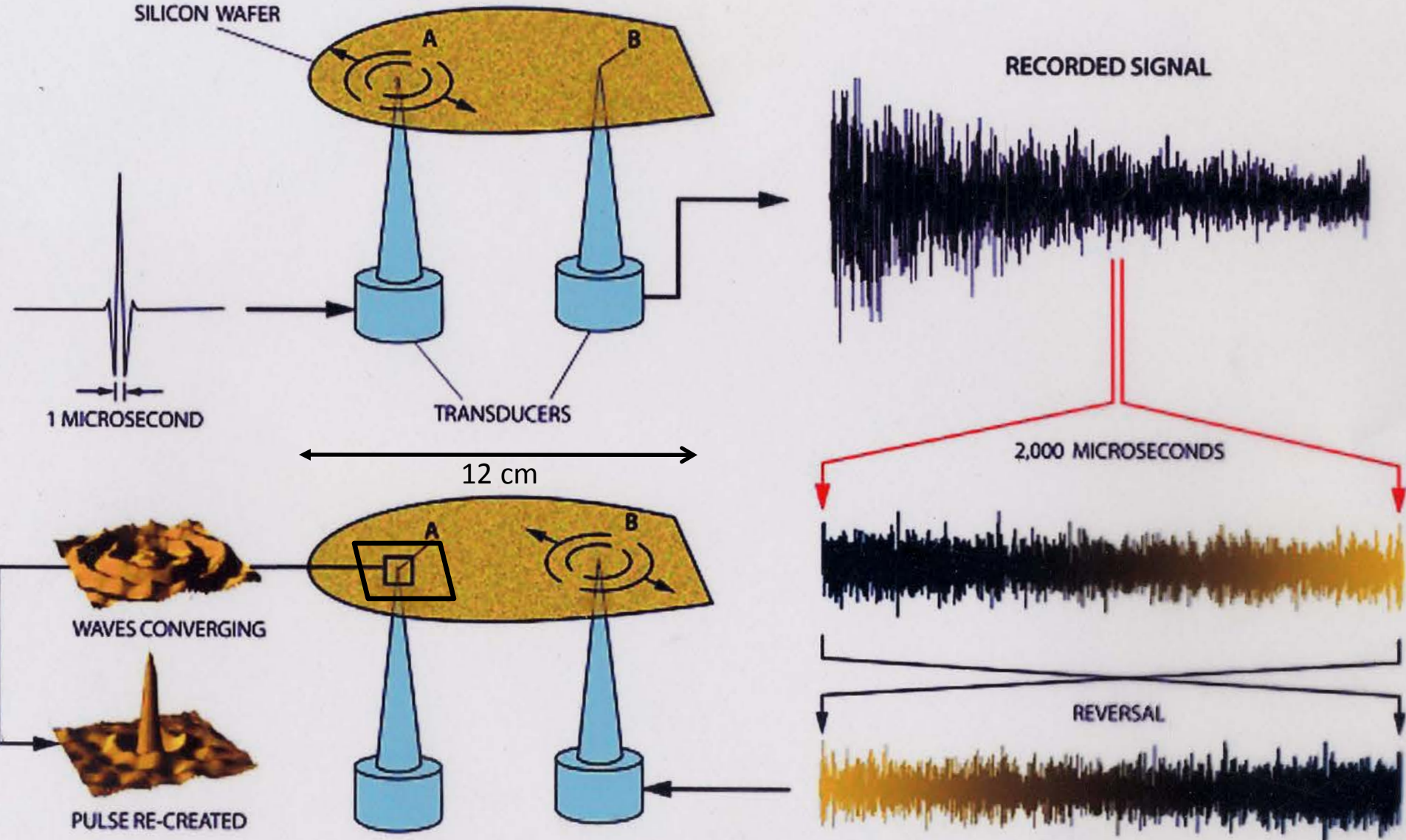
FT



Thoules time, $\delta\tau = L^2/D \sim 150 \mu\text{s}$ \longrightarrow $\delta\omega = 8 \text{ kHz}$ $\Delta\omega/\delta\omega = 150$

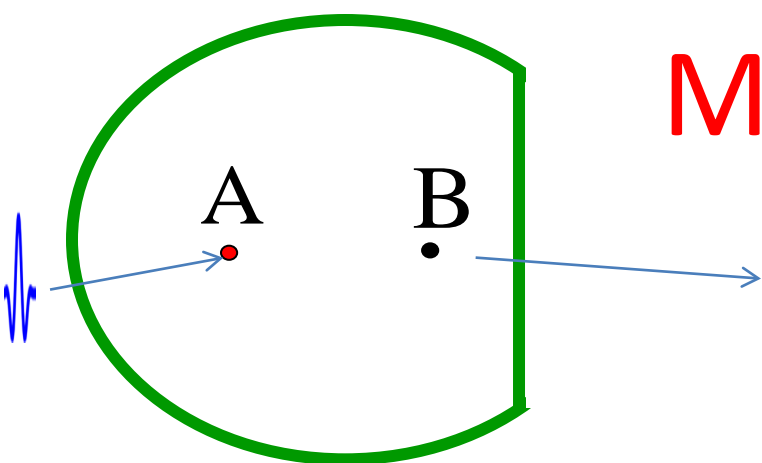
How to transform a closed cavity in a lens

A one channel time-reversal antenna

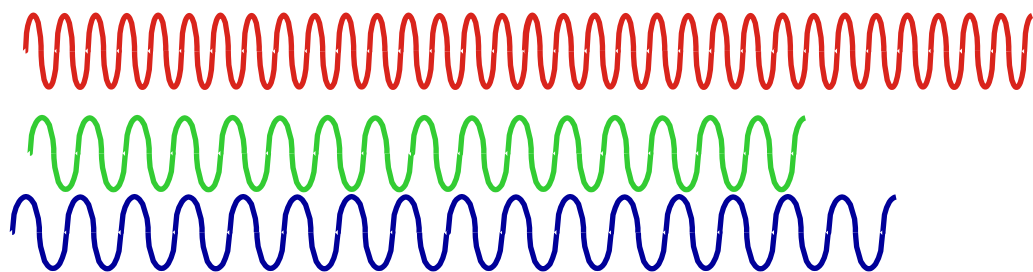


C. Draeger, M. Fink, « One channel time-reversal of elastic waves in a chaotic 2D-silicon cavity »
Physical Review Letters, **79** (3), 407-410, 1997

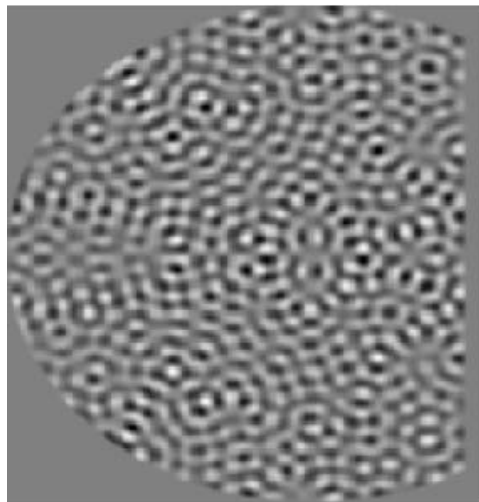
Multi-modal Focusing



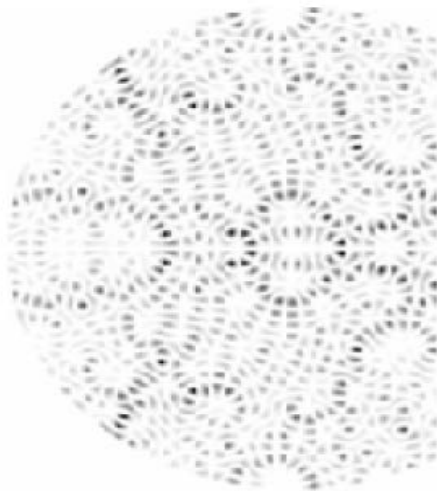
The signal recorded in B is the summation of all the eigenfrequencies excited by the source in A



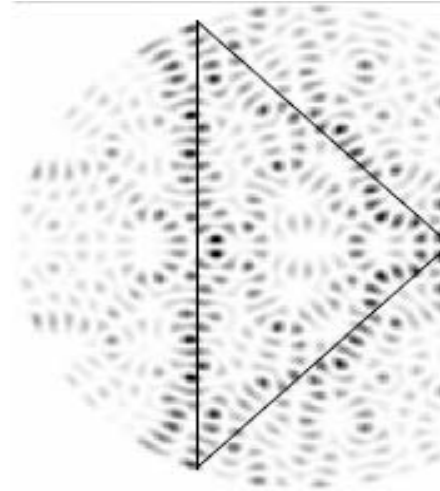
ω_1 mode 1



ω_2 mode 2



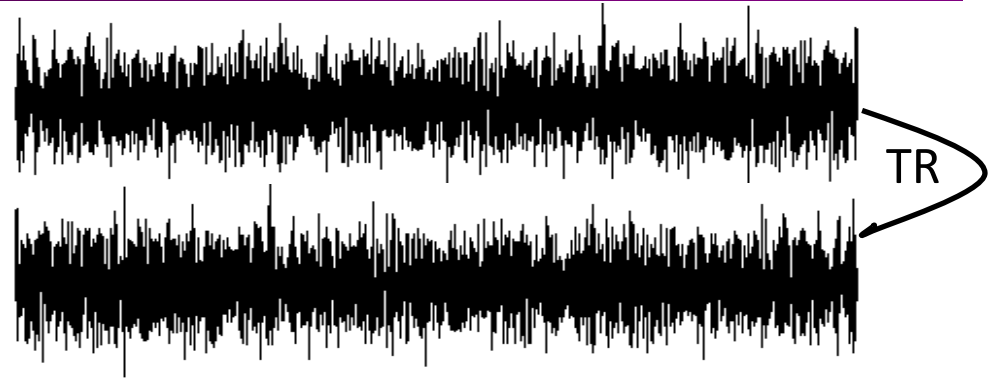
ω_3 mode 3



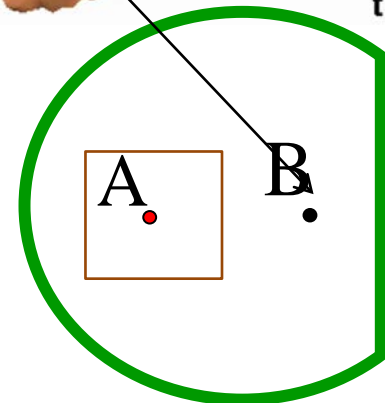
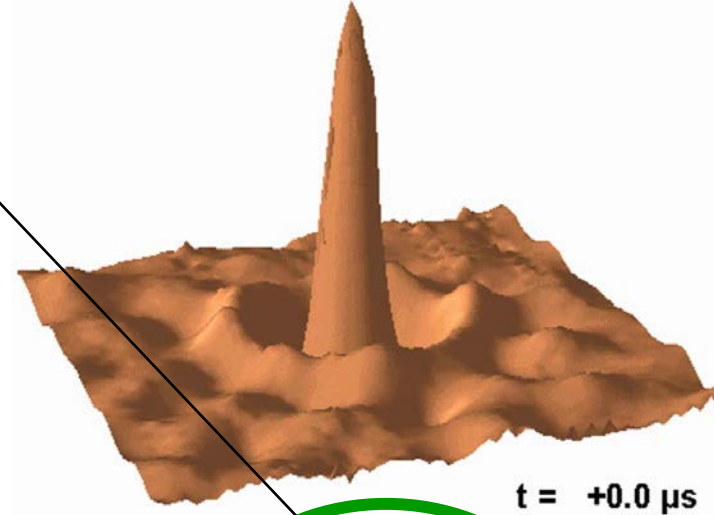
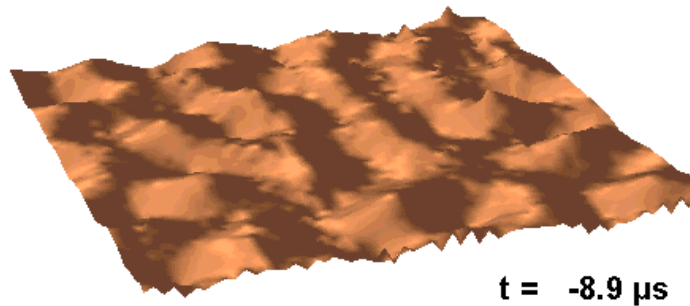
The time-reversed wave optically detected (surface wave)

A 2 ms duration signal transmitted by point B :

A time Hologram

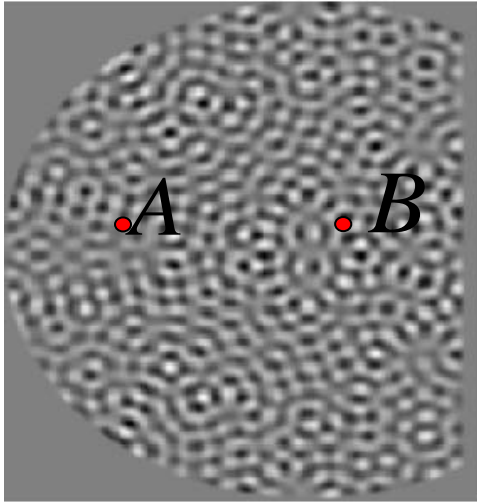


Displacement field recorded on a square 15 x 15 mm²

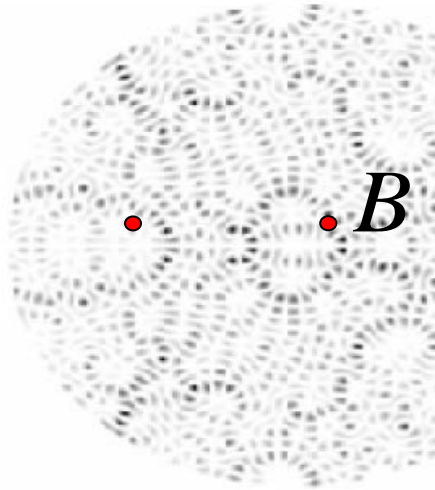


Multi-modal Focusing

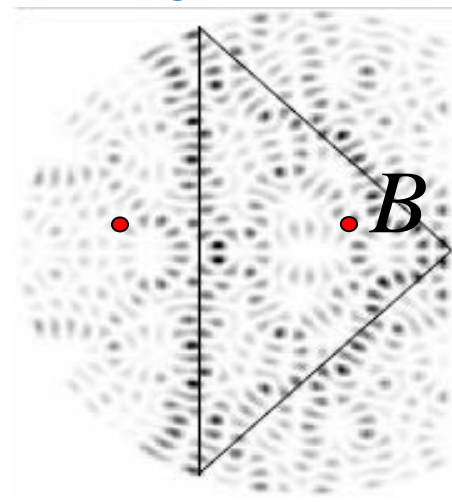
ω_1 mode 1



ω_2 mode 2



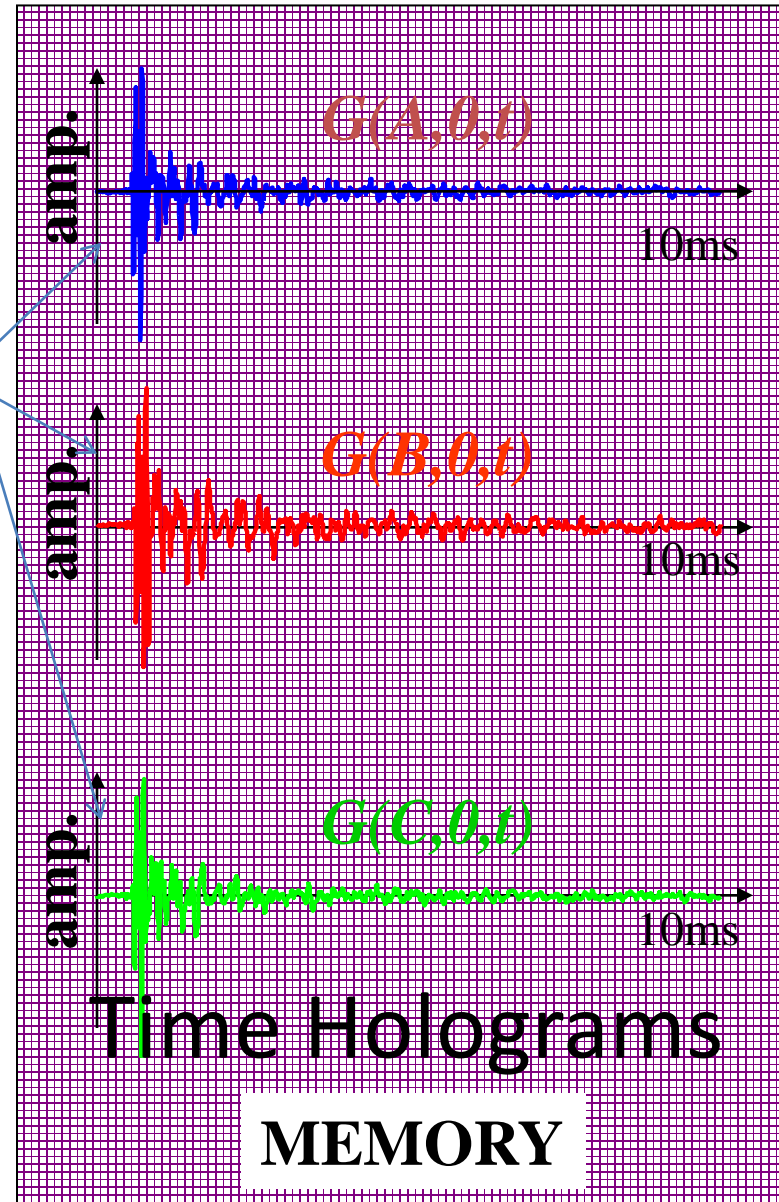
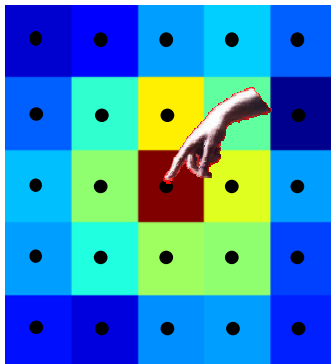
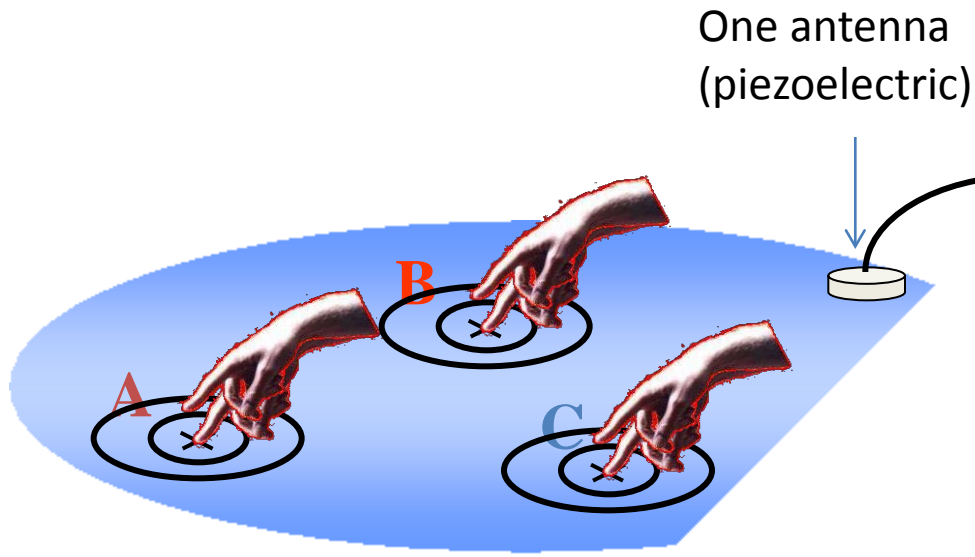
ω_3 mode 3



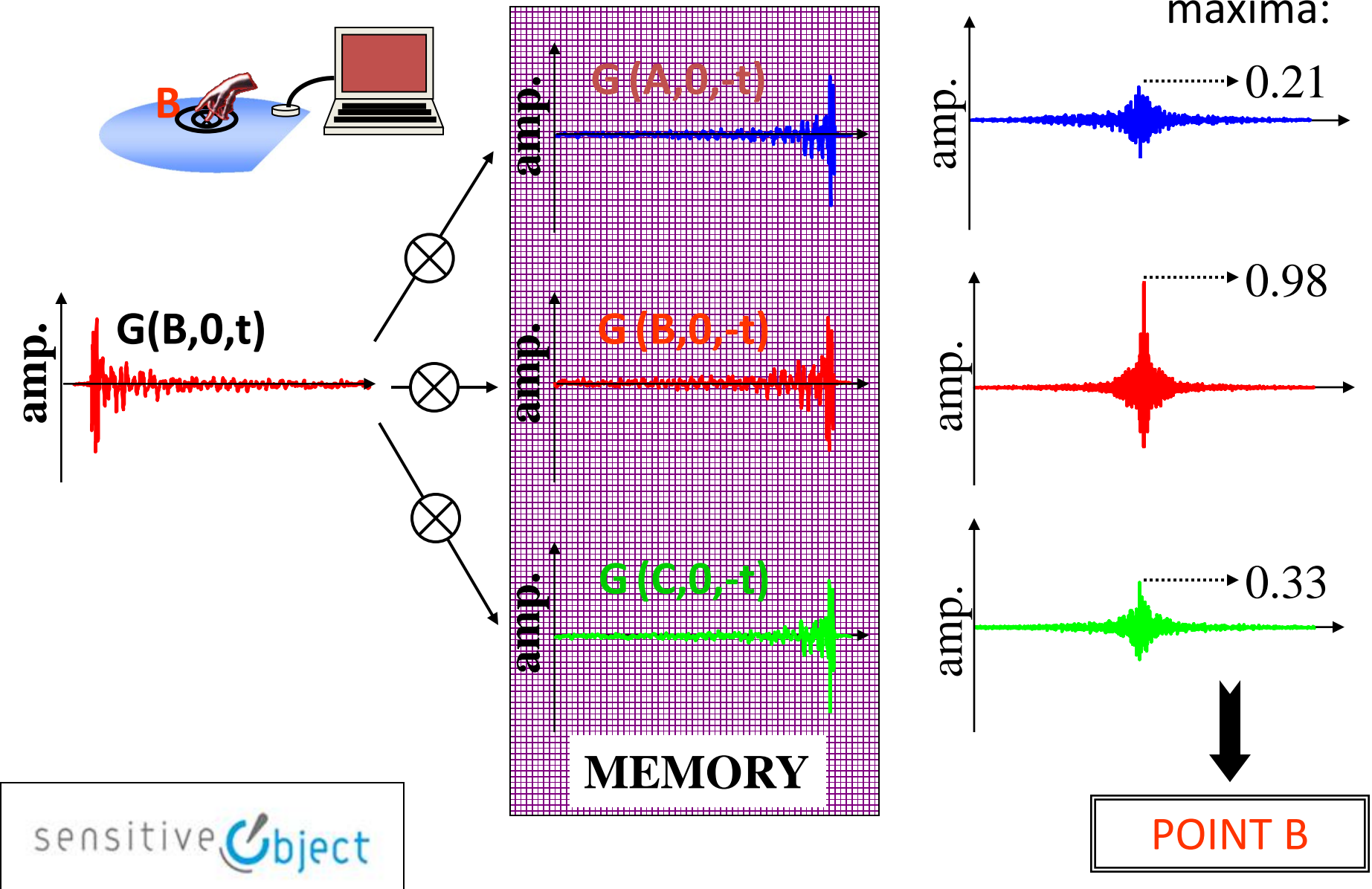
During the time-reversed step from point B, 400 uncorellated eigenmodes are excited with perfect phase matching at point A

The size of the focal spot is of the order of the size of one modal cell = $\lambda/2$

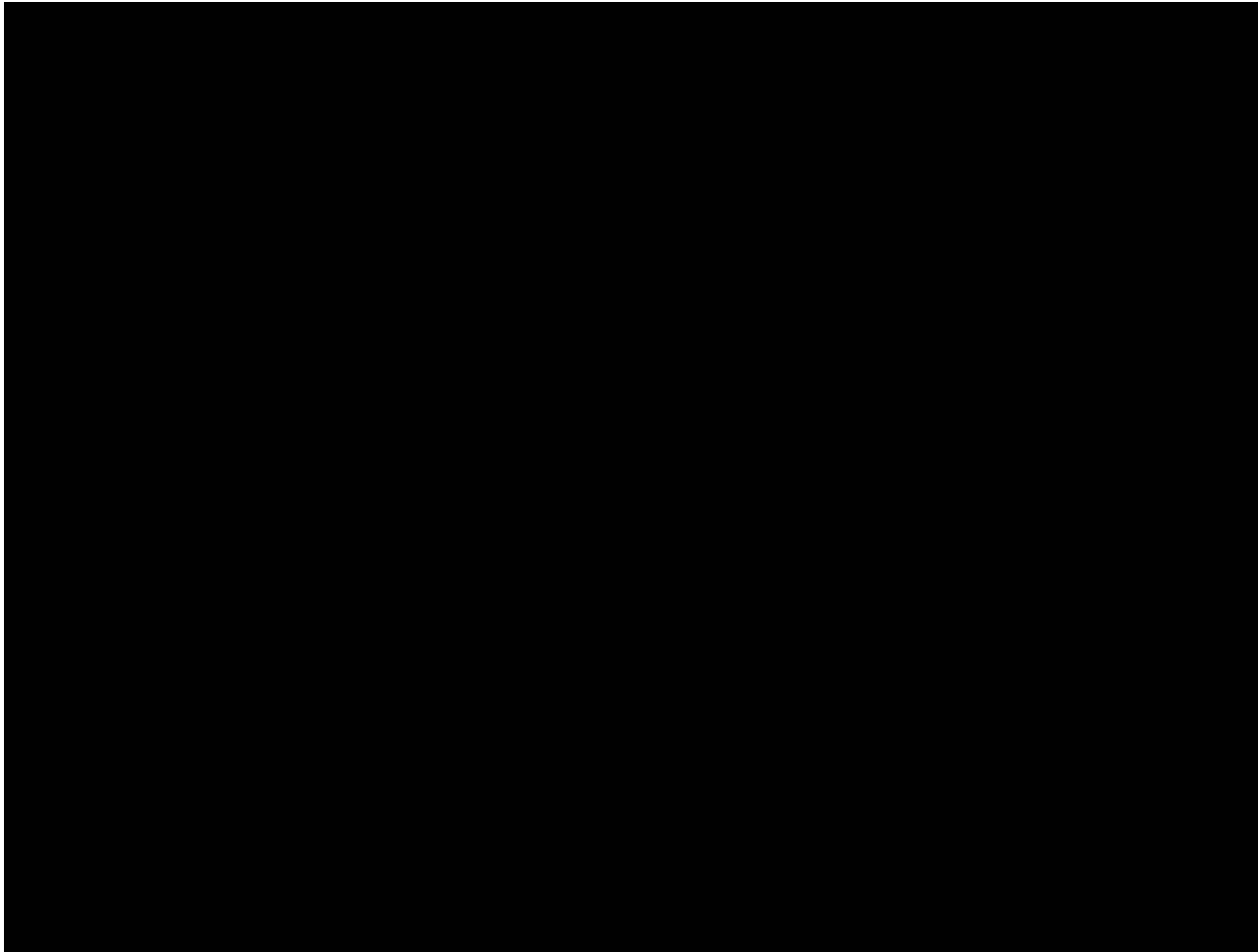
Tactile Objects



Source localisation by cross-correlation, mimicking a time-reversal experiment in the computer



Products : Tactile Objects



The noise radiated by the finger is enough to follow the trajectory

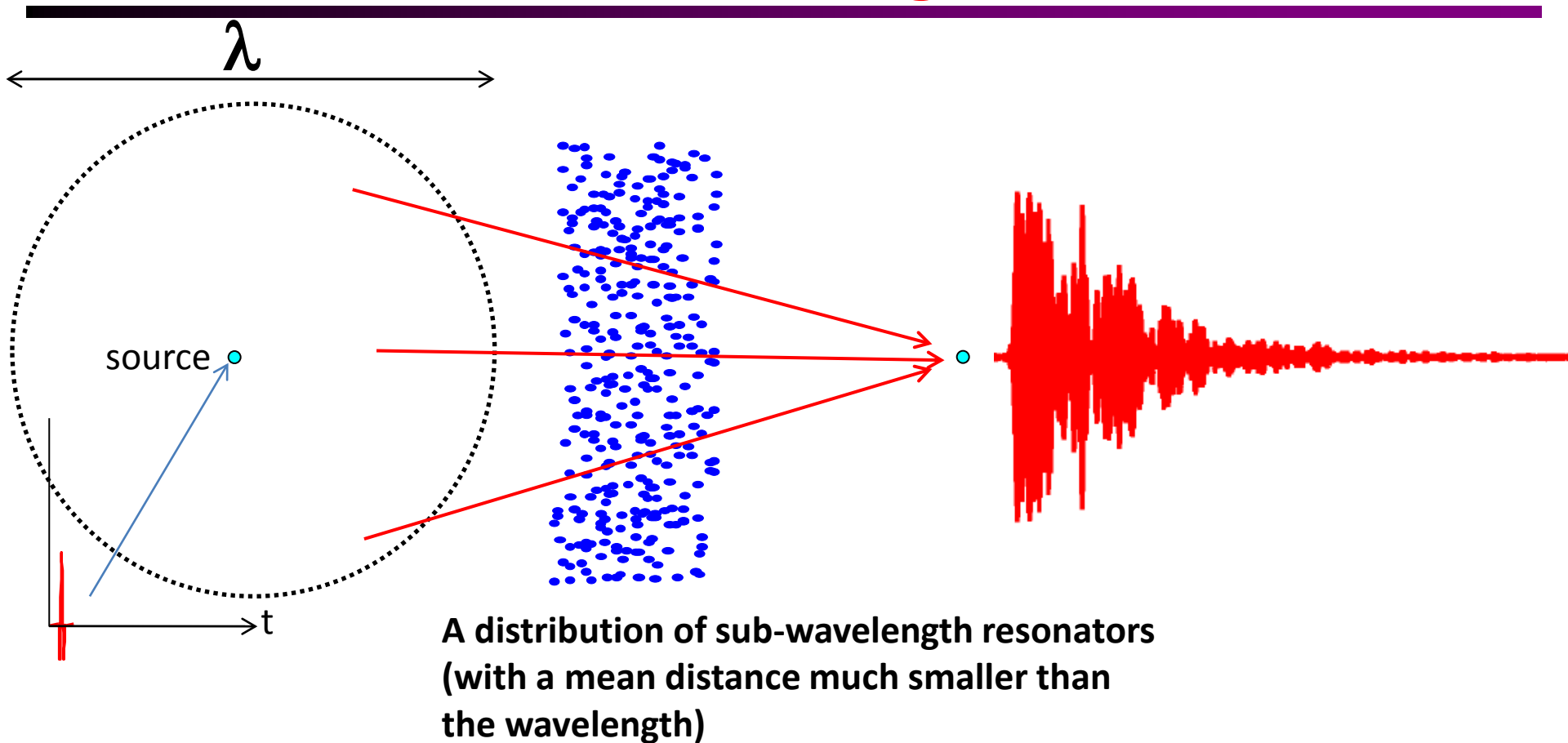
Sensitive Object , 35 employees Now Tyco Electronics

Subwavelength Focusing with TR

Two different approaches :

- distribution of sub-wavelength resonators in the near field of the source
- the sink (a perfect absorber), introducing dissipation

How to build a dispersive medium with subwavelength scale ?



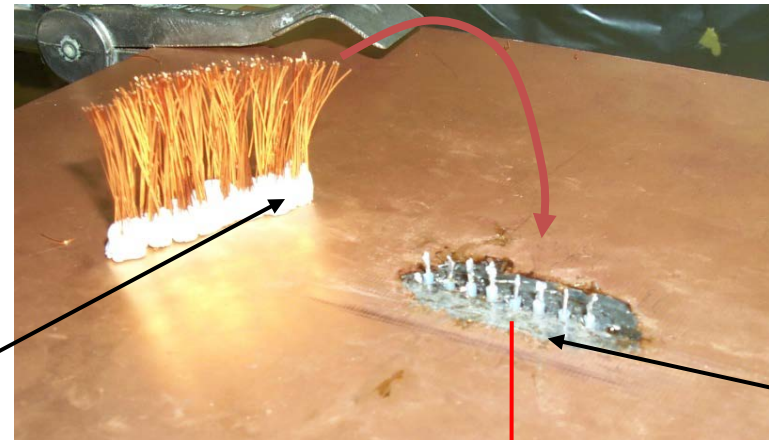
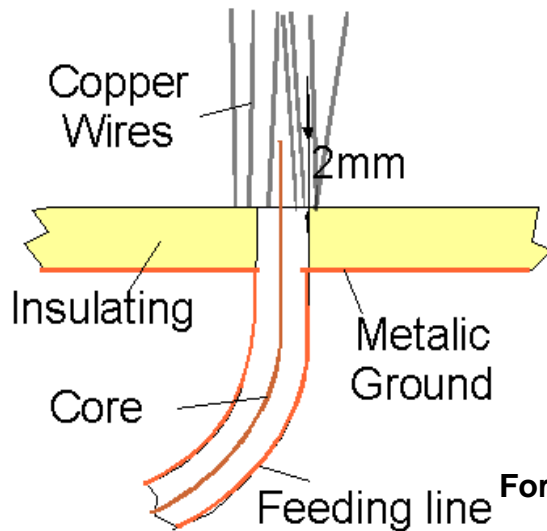
Near field and multiple scattering

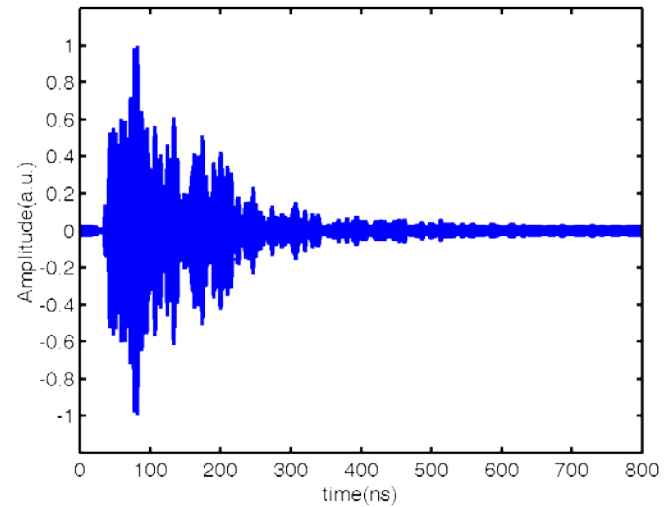
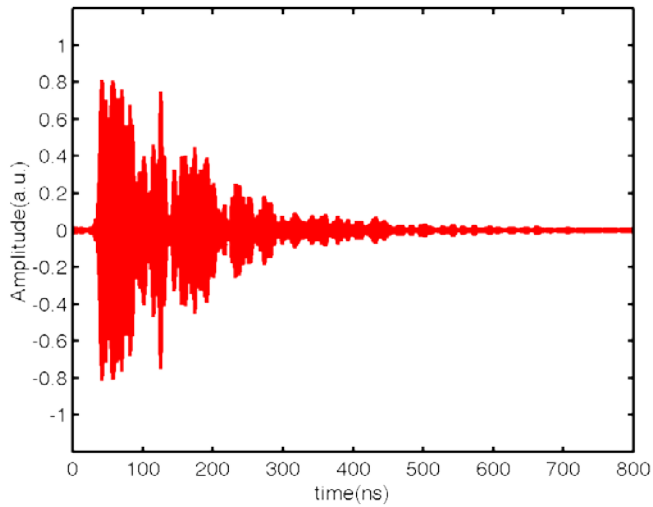
Frequency Diversity

Super-resolution

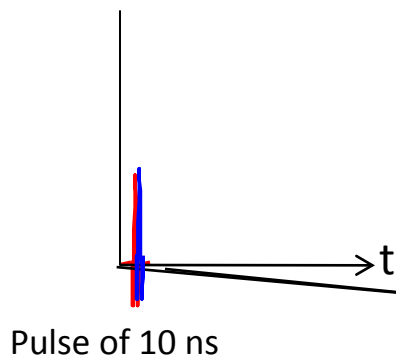
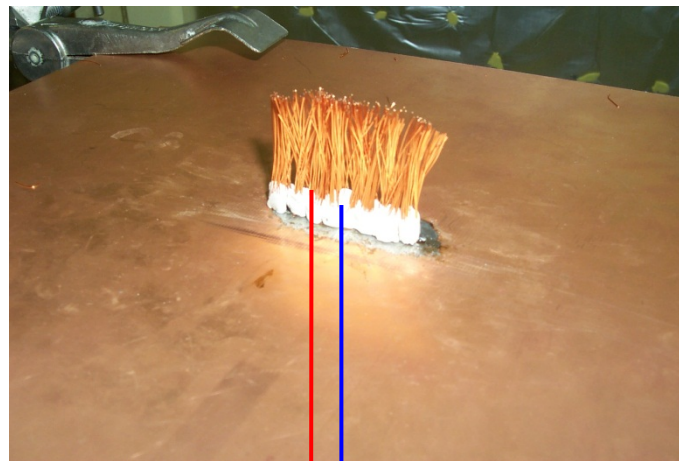
Electromagnetic Waves : wavelength 12 cm
at 2.44 GHz- WiFi Frequency)

You have to locate in the near field of the source (less than a wavelength) a distribution of resonating scatterers



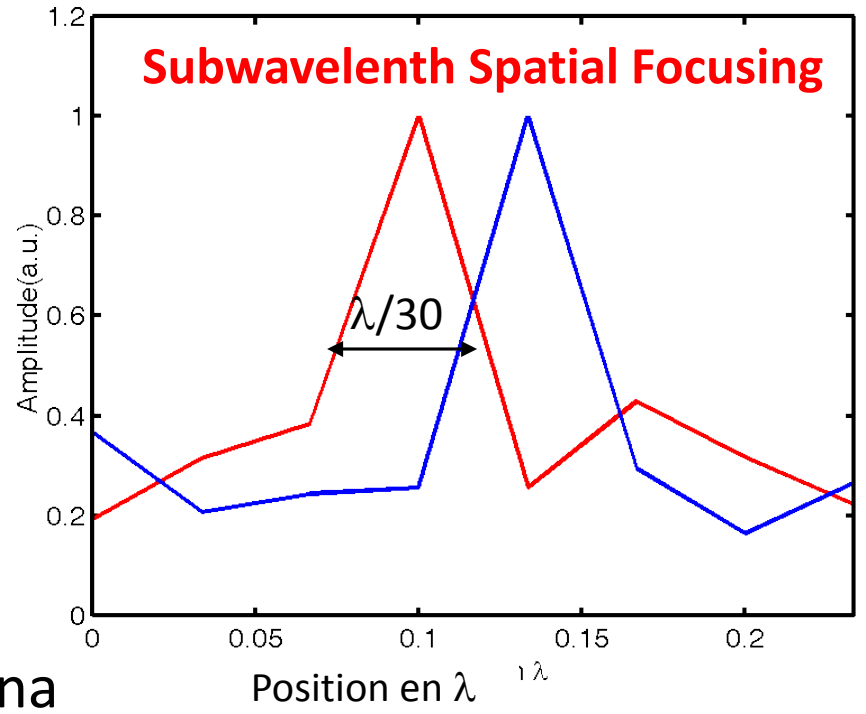
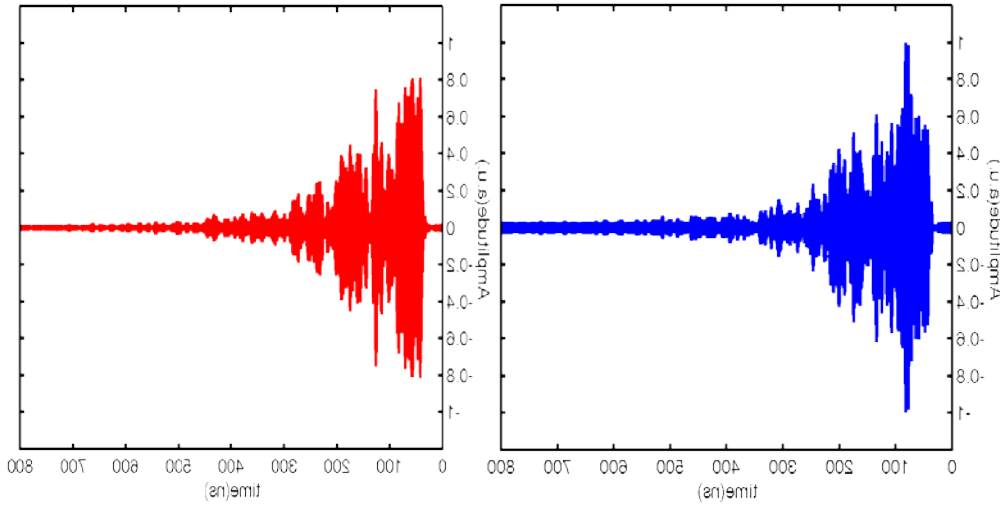


**Signal recorded in the far field
by one TRM antenna**



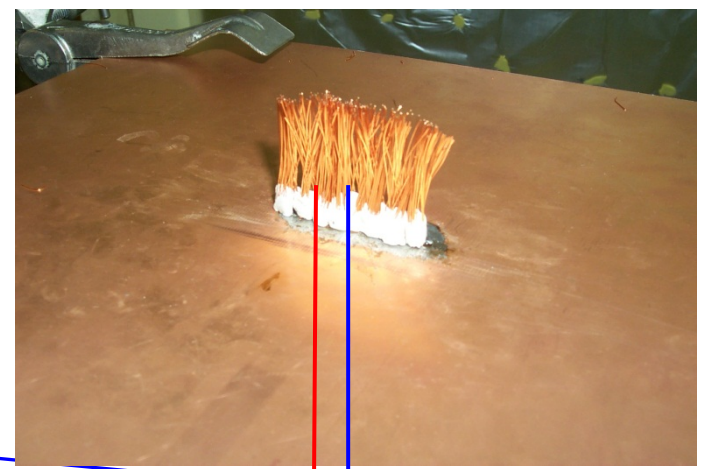
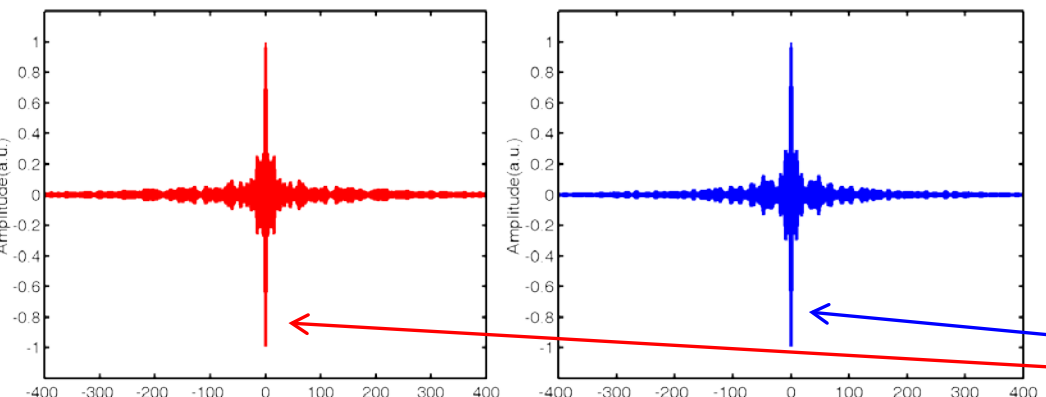
$4\text{mm} = \lambda/30$

Sub-wavelength resolution by a far field TRM



Time-reversed signals transmitted and observed by the red or blue wire antenna

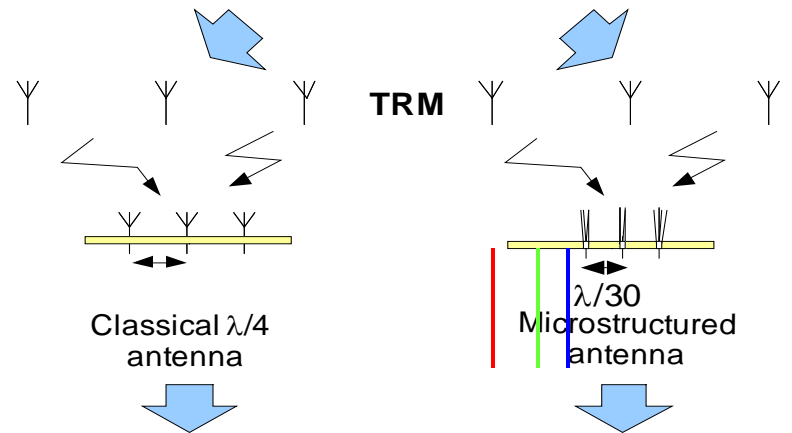
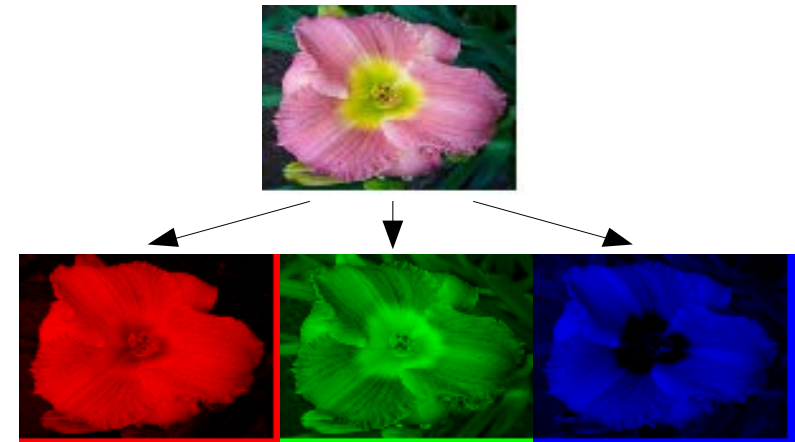
Time-Focusing



Telecommunications

3 bitstreams (RGB) with 50 Mbits/s.
The total rate transfer is 150 Mbits/s.

The TRM was made of 3 antenna
with 2.45 GHz center frequency
and 180 MHz bandwidth



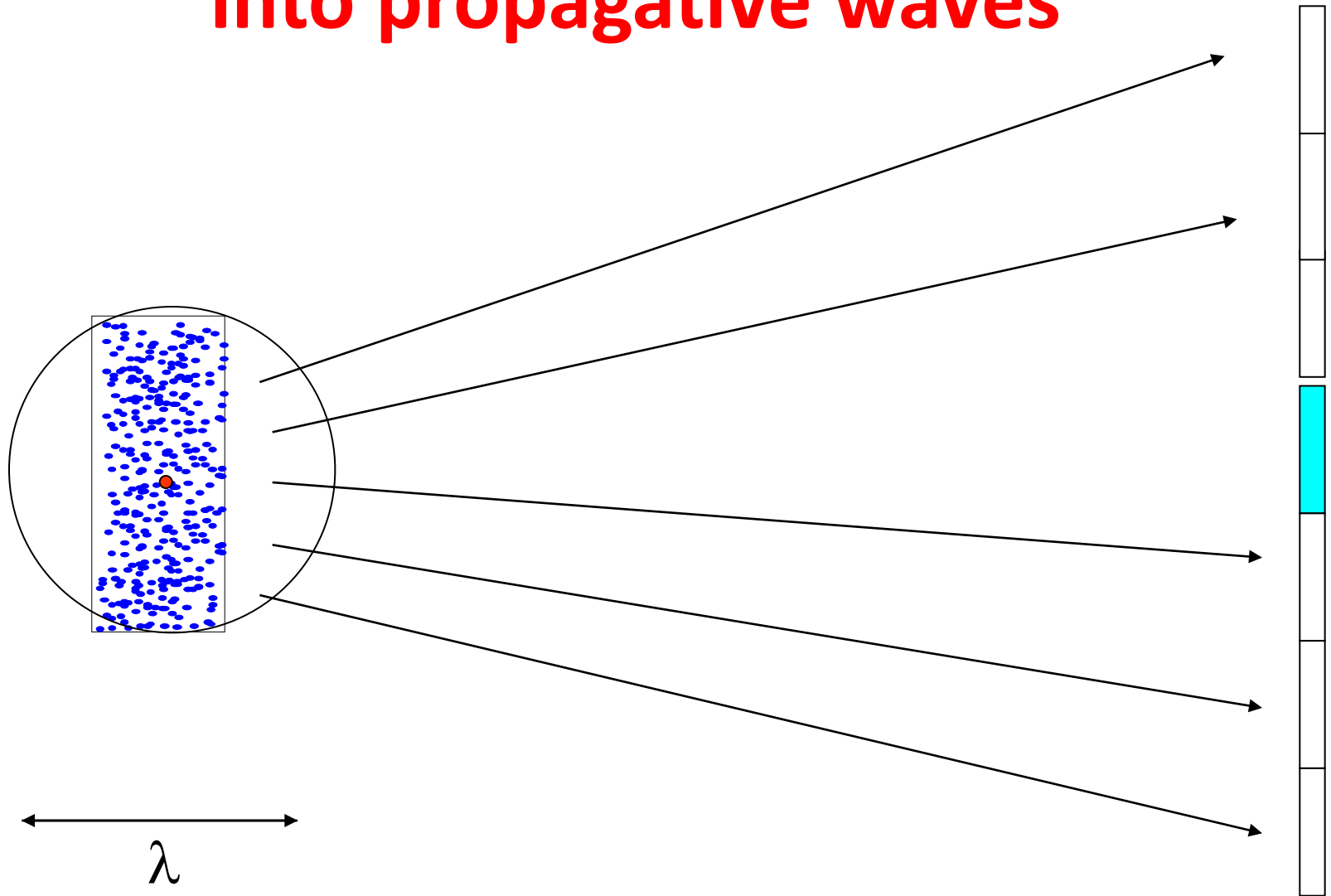
(a)



(b)

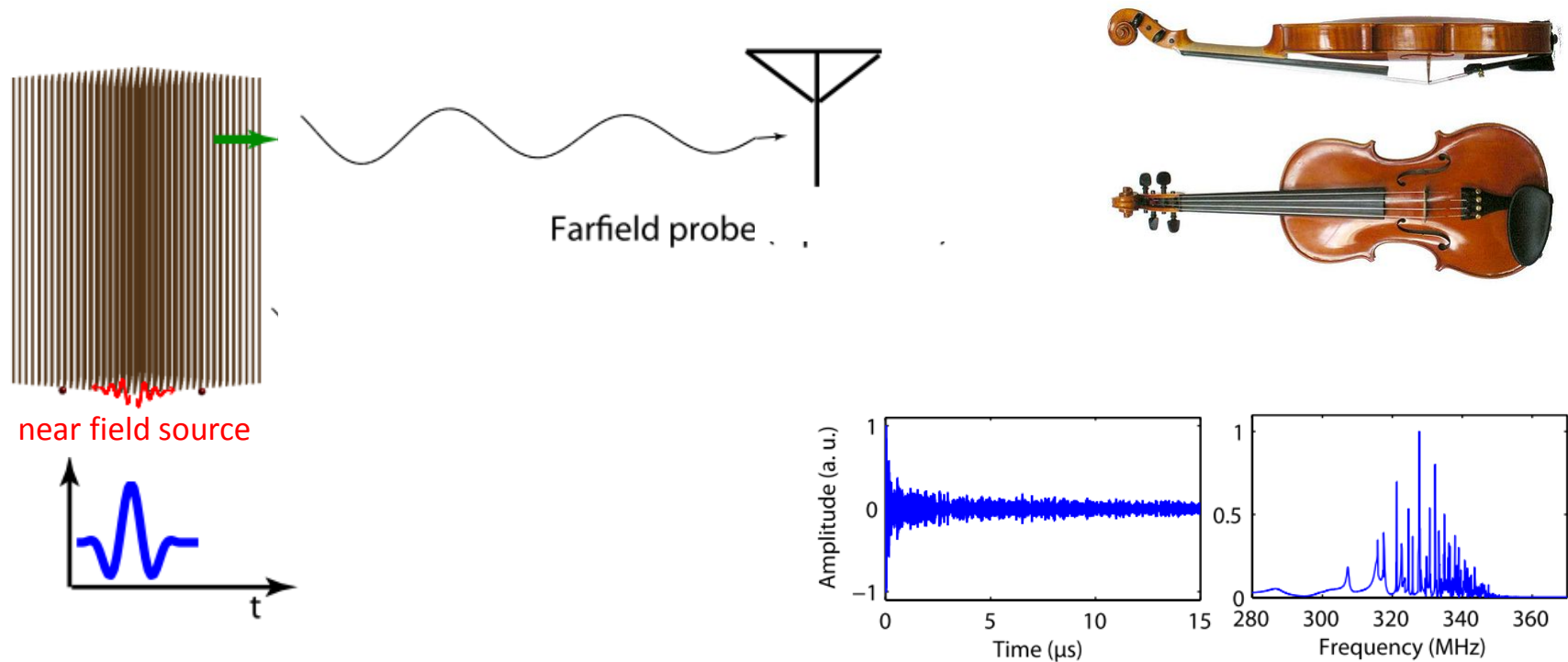
Start-up Time-Reversal Communications
(40 employees, now Bull)

The evanescent field is converted into propagative waves



What mechanism transforms evanescent waves into propagative waves ?

The multi-modal Board



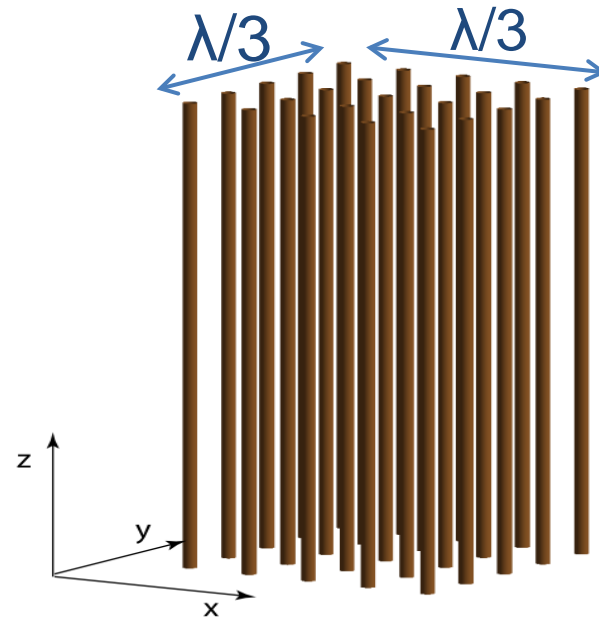
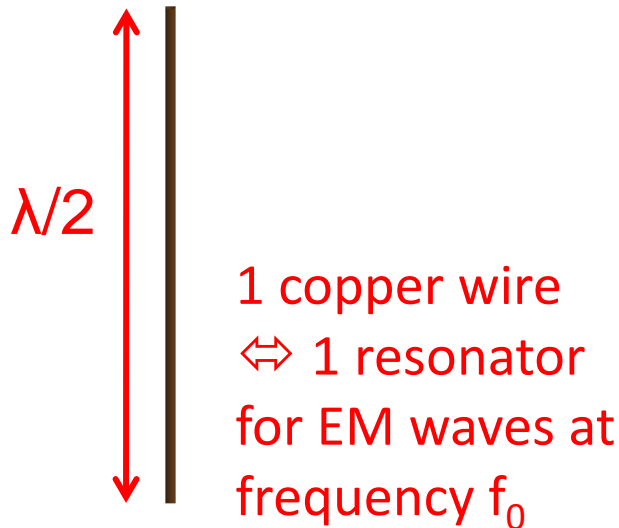
F. Lemoult, G. Lerosey, J. de Rosny, M. Fink, Phys. Rev. Lett., 104, p 203901, 2010

F. Lemoult, M. Fink, G. Lerosey, Waves in random and Complex media, 21 (4) 614-627, 2011

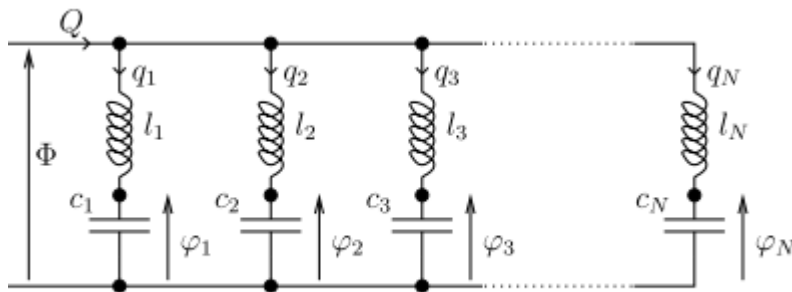
F. Lemoult, M. Fink, G. Lerosey, Waves in random and Complex media, 21 (4) 591-613, 2011

Two key points :

1. The medium is made of coupled subwavelength resonators
2. The medium must be of finite lateral size

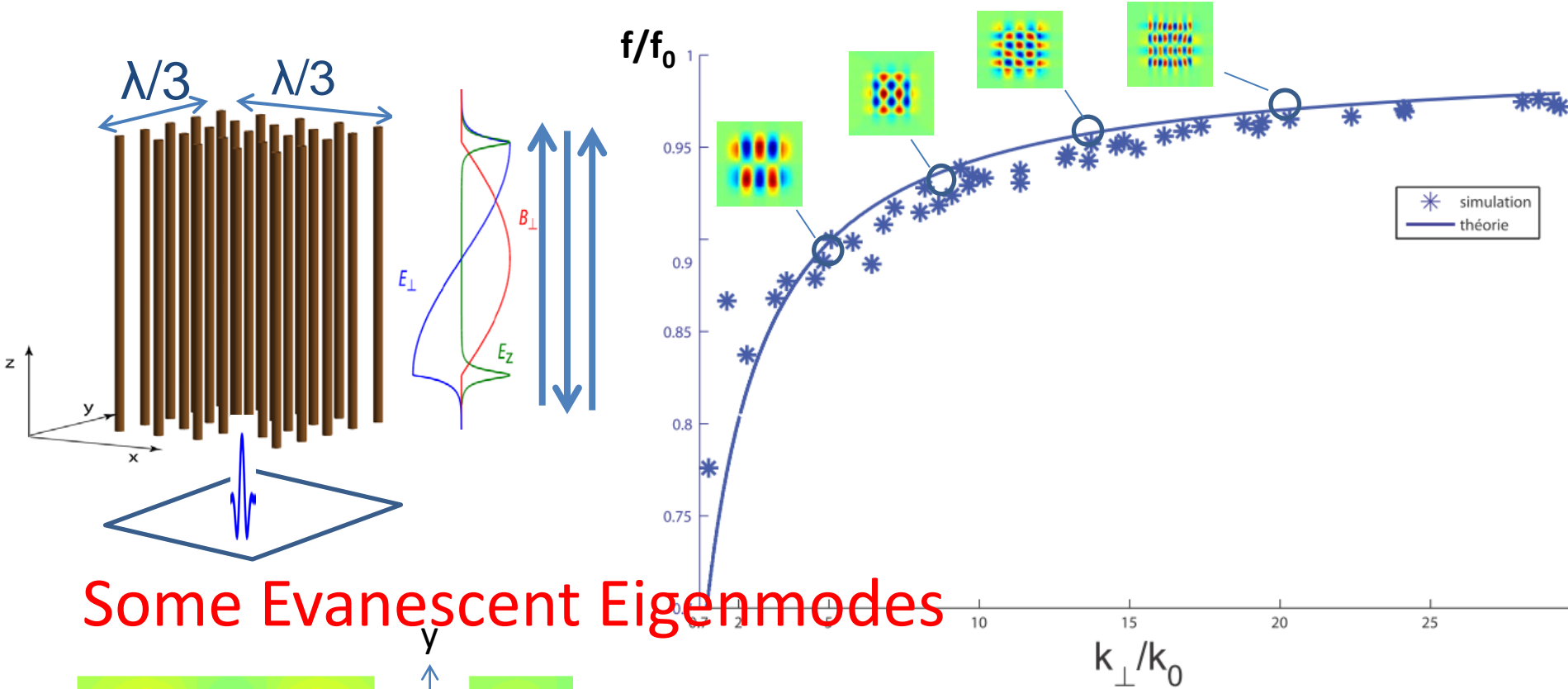


An example : $(\lambda/80)$ spaced resonators in a periodic structure (or random)

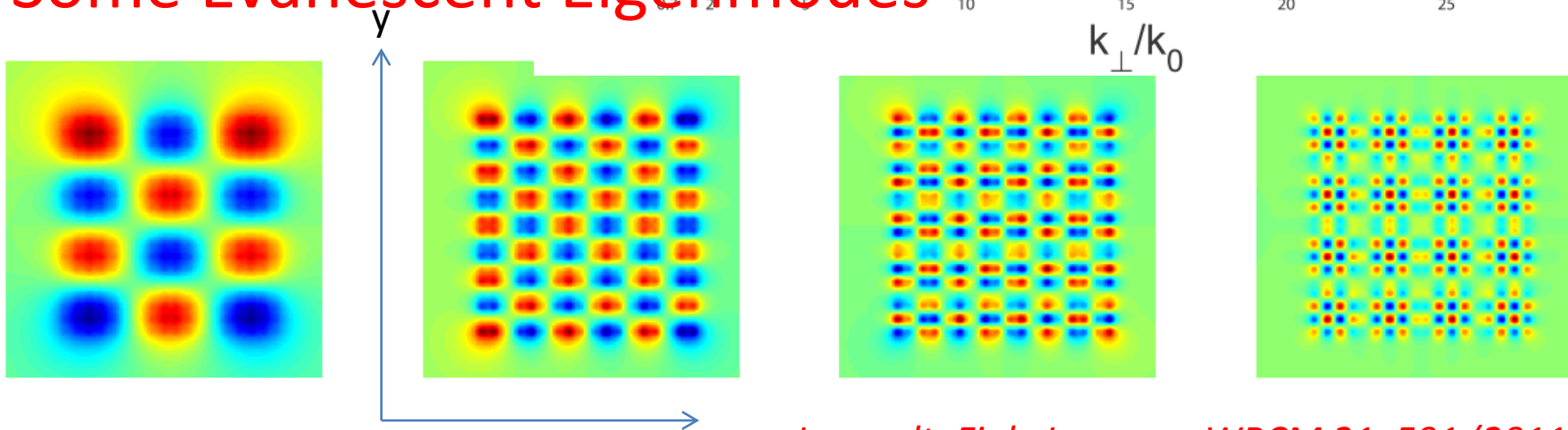


**N electrical coupled oscillators
 $\Rightarrow N$ eigenmodes and
 N eigenfrequencies**

Dispersion of TEM Waves

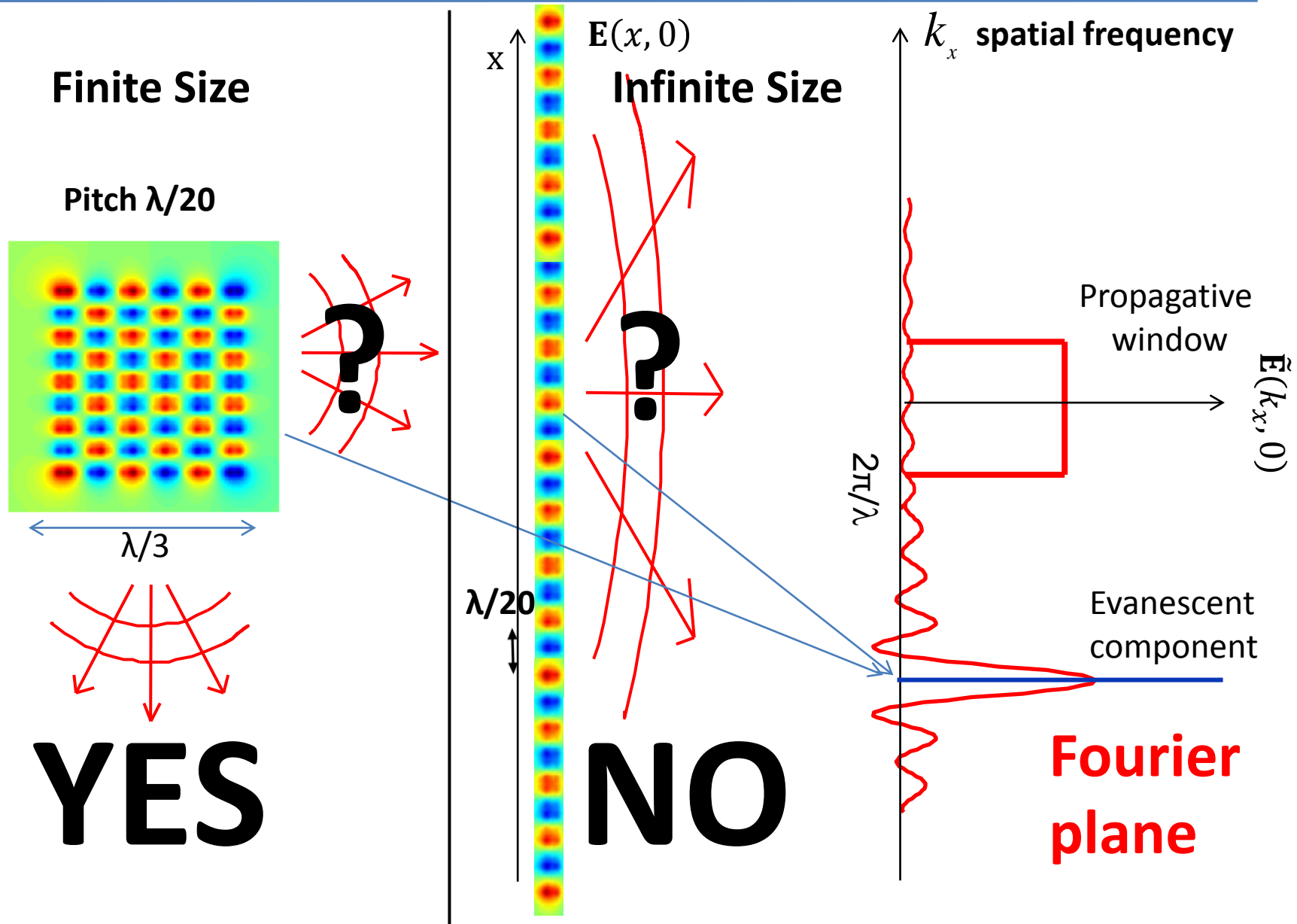


Some Evanescent Eigenmodes



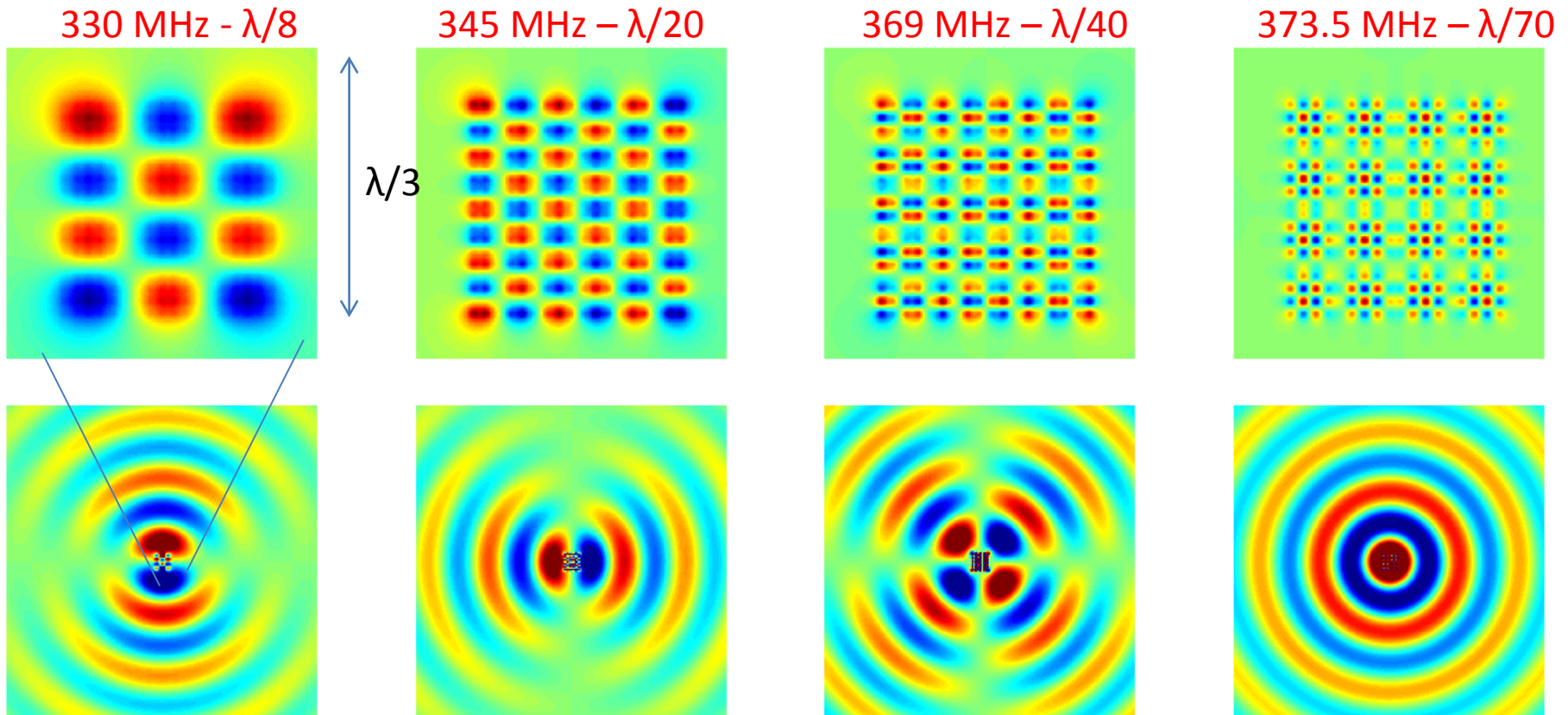
Why evanescent eigenmodes can radiate in the far field ???

Because the wire medium is bounded



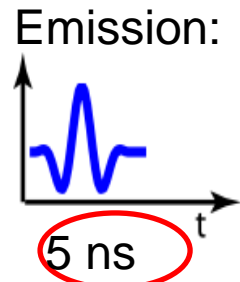
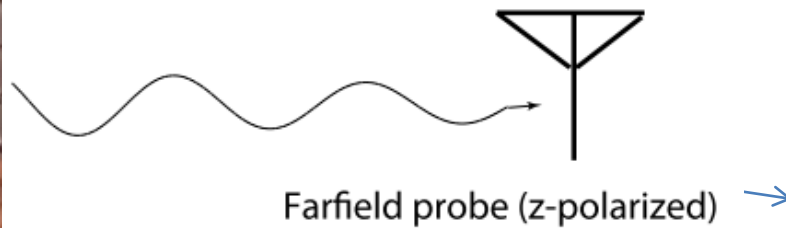
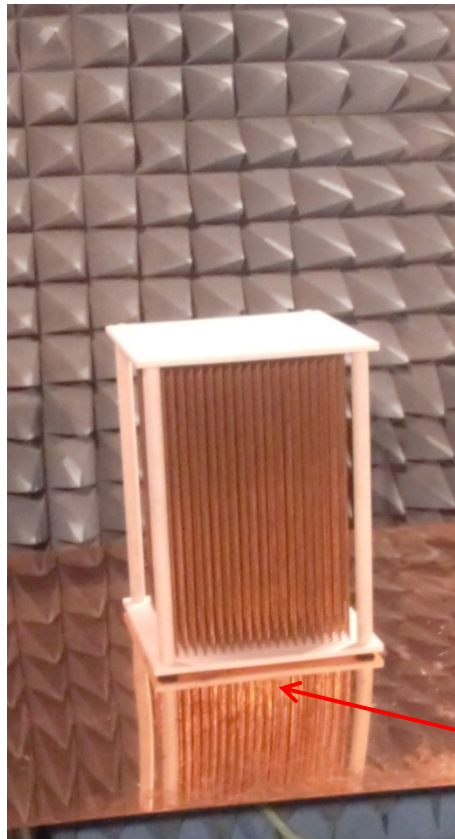
Is it Efficient ? Yes : Resonant amplification

For $f_0 = 375$ MHz

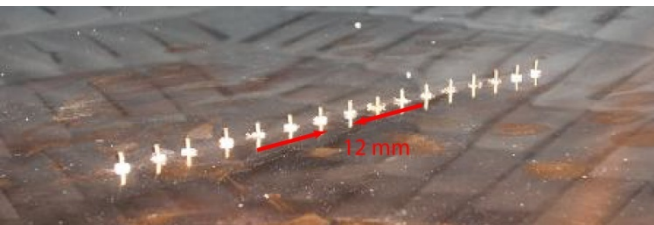
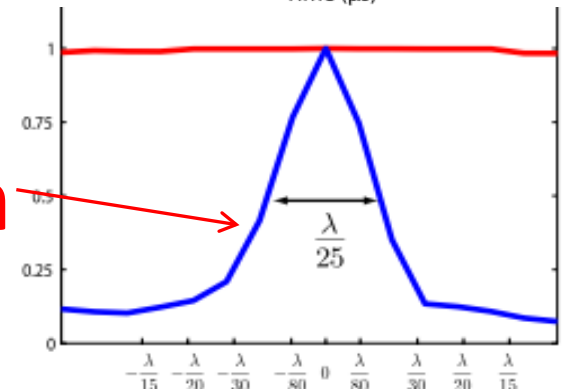
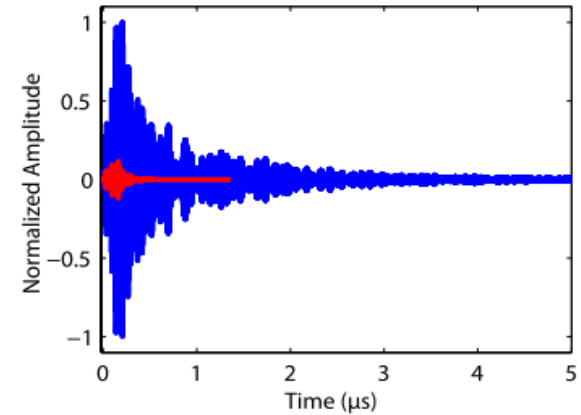
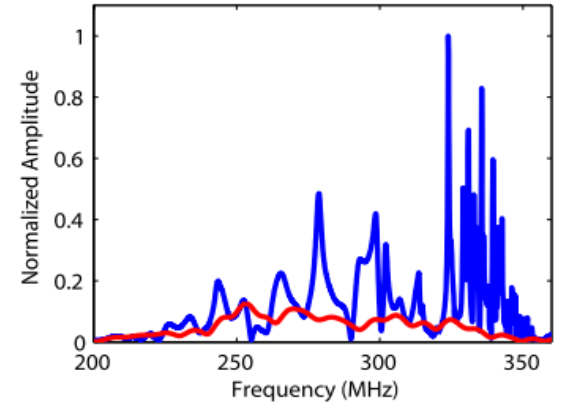


Resonant Amplification : When a mode radiates slowly, the energy is trapped and there is a strong resonant amplification inside the structure: **Purcell Effect**

Subwavelength focusing from the far field



Subwavelength Focusing



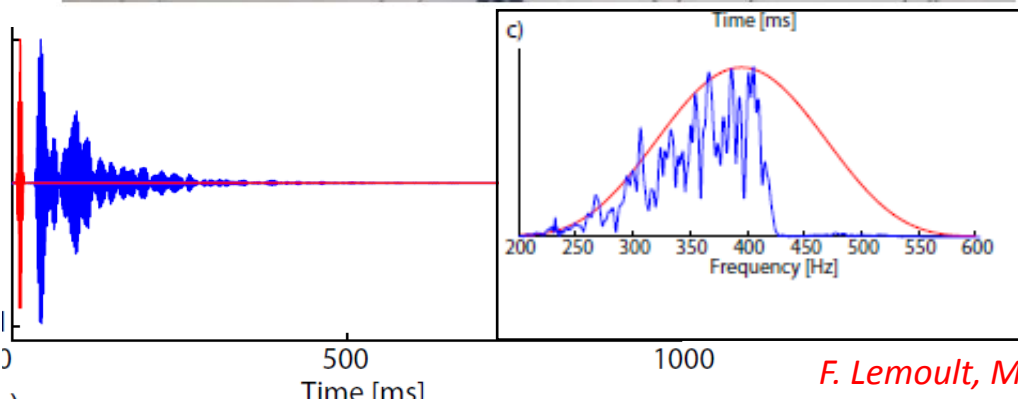
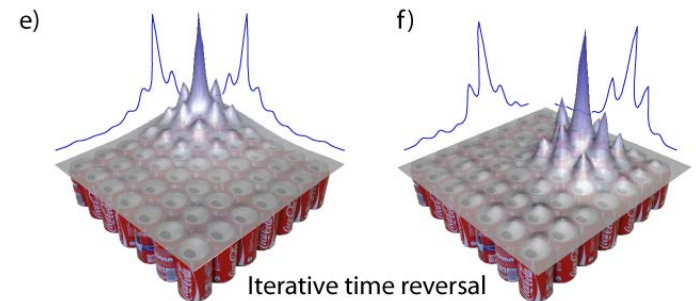
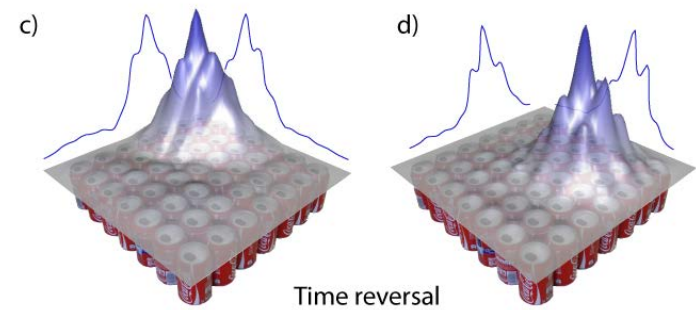
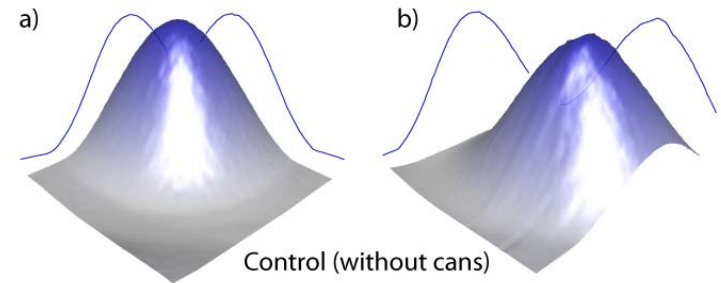
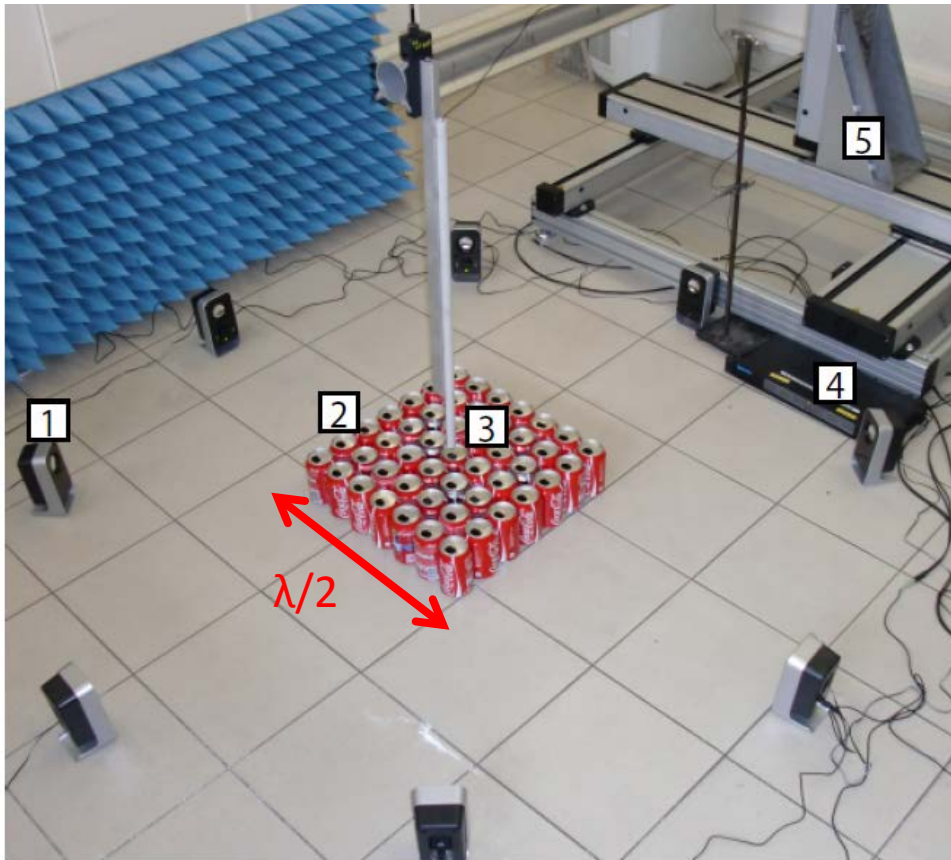
Sub-wavelength acoustic resonators



Resonance in air @ 420 Hz ($\lambda=0.8\text{m}$) , Helmholtz sub- λ resonator (diameter = 6,5 cm), Low loss

F. Lemoult, M. Fink and G. Lerosey, Phys. Rev. Lett. 107, 064301 (2011).

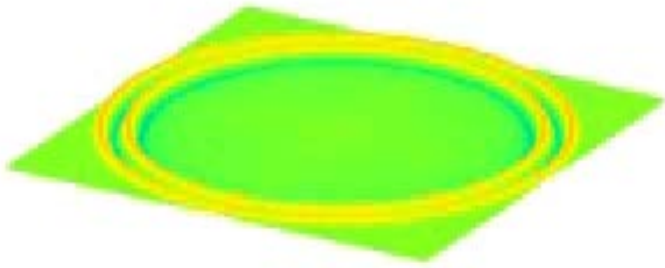
Sound Focusing and Soda Cans



Another Way to Super-Resolution

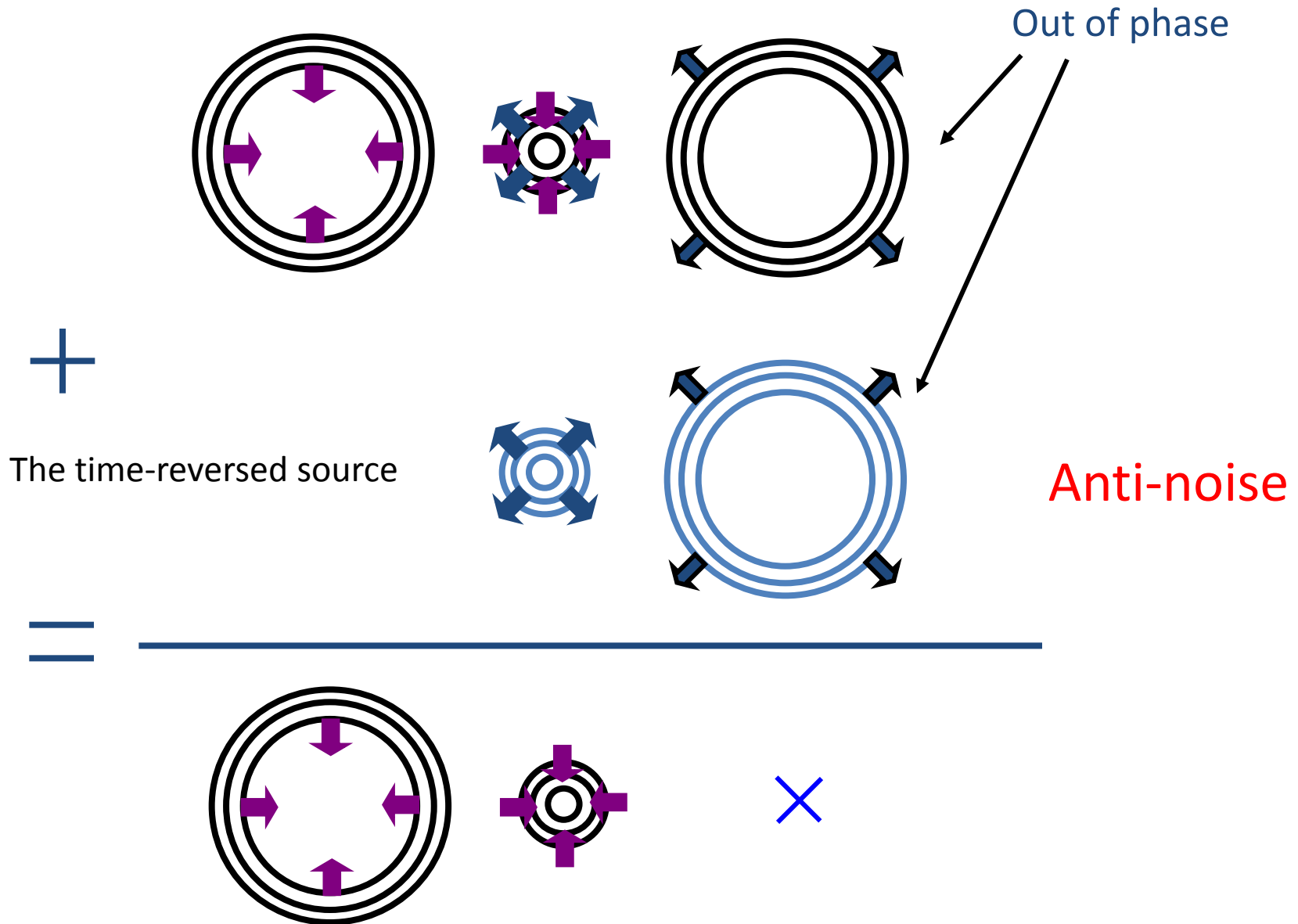
Introducing the Sink - a Perfect Absorber -

Origin of the diffraction limit in free space



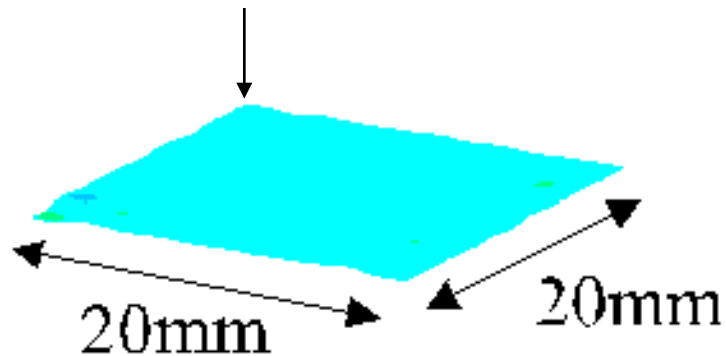
In free space, we cannot create only a converging wave, even with 4π aperture lens (a TR cavity or a Maxwell fish eye)

Principle of an active acoustic sink

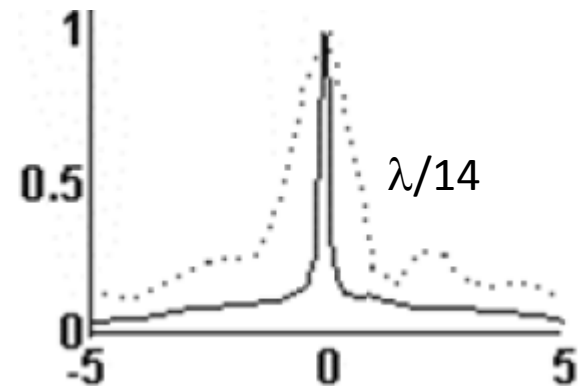
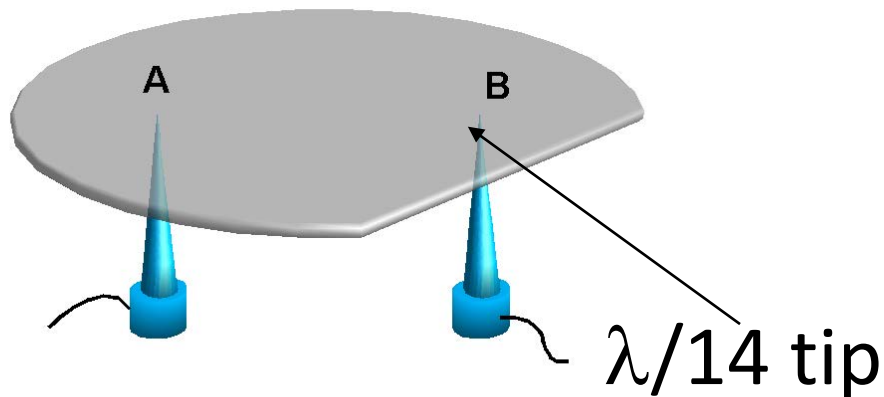
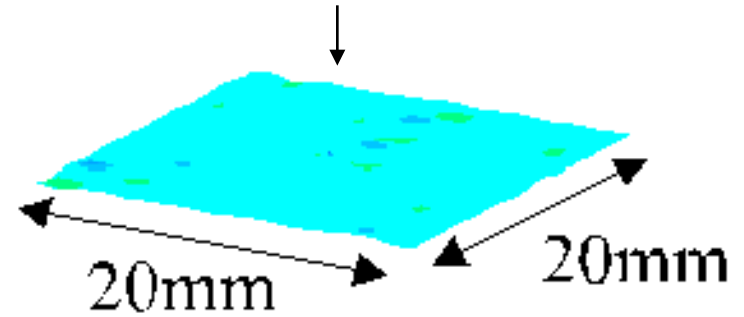


Experimental Acoustic Sink

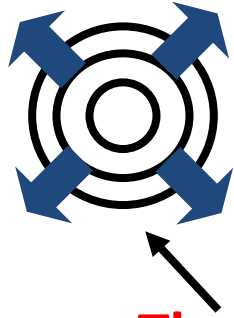
Wavefield Time Reversal



WaveField and Source Time Reversal



To create only a **converging wave** requires to time-reversed also the source : the Sink (drain)

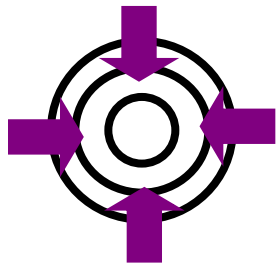


The causal field : **diverging wave**

$$\left(\Delta - \frac{1}{c^2} \frac{\partial}{\partial t^2} \right) \varphi(\vec{r}, t) = f(t) \delta(\vec{r} - \vec{r}_0)$$

Point-like source

To create a perfect anti-causal field : a **converging wave** only
 You have to replace t by $-t$ in the wave equation



Converging wave

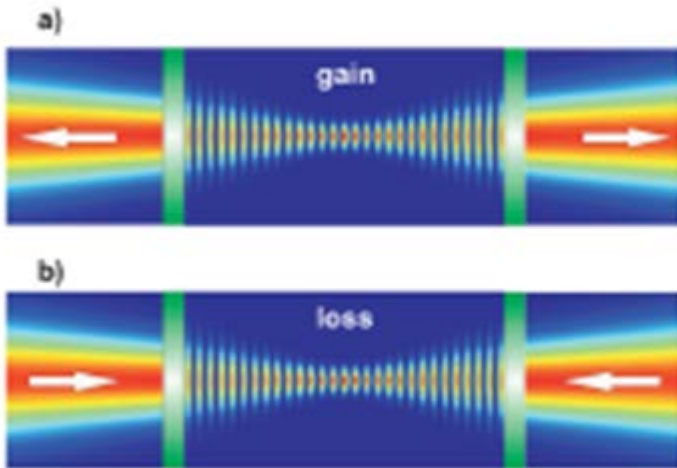
$$\left(\Delta - \frac{1}{c^2} \frac{\partial}{\partial t^2} \right) \varphi(\vec{r}, -t) = f(-t) \delta(\vec{r} - \vec{r}_0)$$

The time-reversed source is modulated by $f(-t)$

Is it possible to build a passive Sink ? Introducing Dissipation (loss)

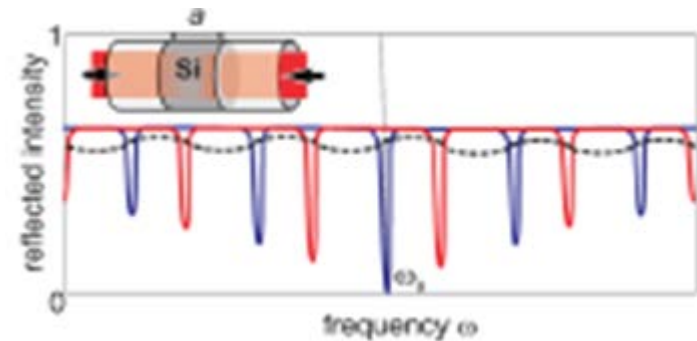
The coherent perfect absorber : the **time-reversed Laser**
: a sink **for monochromatic wave** (Loss is the time reversal of Gain)

PRL 105, 053901 (2010) Selected for a Viewpoint in Physics PHYSICAL REVIEW LETTERS week ending 30 JULY 2010



Coherent Perfect Absorbers: Time-Reversed Lasers

Y. D. Cheng,* Li Ge, Hui Cao, and A. D. Stone



A Fabry Perot Resonator with gain is transformed in TR operation in a Fabry Perot with loss

Is it possible to build a blackbody (broadband) of size smaller than the wavelengths ?

Time-reversal of visible light

Control of light transmission through opaque scattering media in space and time

Optical time reversal was still a dream 2 years ago...

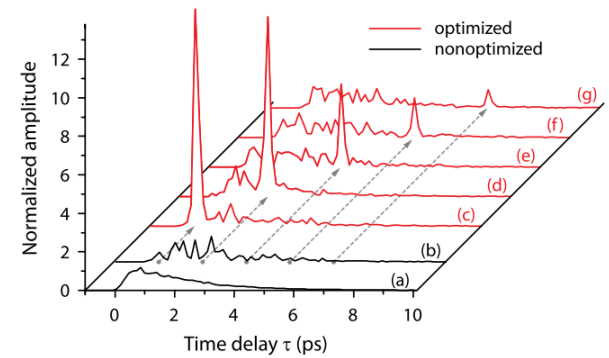
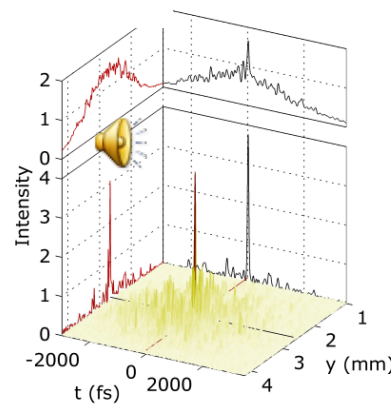
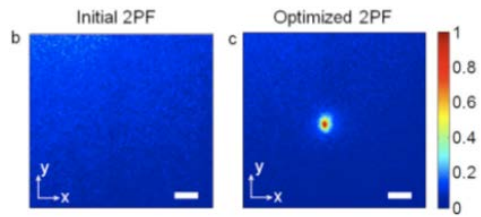
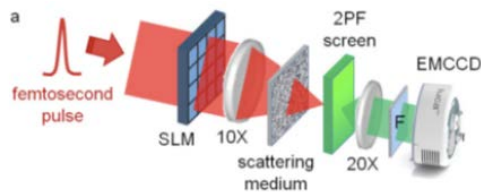
Jochen Aulbach,^{1,2,*} Bergin Gjonaj,¹ Patrick M. Johnson,¹ Allard P. Mosk,³ and Ad Lagendijk¹

Controlled Spatiotemporal Focusing Through Turbid Media

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Shaping speckles: spatio-temporal focussing of an ultrafast pulse through a multiply scattering medium

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Controlling waves in space and time for imaging and focusing in complex media

Allard P. Mosk¹, Ad Lagendijk^{1,2}, Geoffroy Lerosey³ and Mathias Fink³